# Orthometric Height Derivation from GPS Observations

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**Key words**: Orthometric height, leveling, gravimetric geoid, GPS/leveling geoid, collocation.

## ABSTRACT

In order to obtain Helmert orthometric height of a point which already has GPS coordinates, difference between the gravimetric geoid and the GPS/leveling geoid heights at 187 points that are uniformly spread all around the country have been modeled by least squares collocation (LSC) and 3'x 3' size grid output file have been formed.

Initially, FORTRAN IV code and, for ease of use, also a MATLAB script has been prepared by using the Graphical User Interface (GUI) tool kit of MATLAB 5.3 software.

Several tests have been applied with different data sets. Accuracy of the height determination have been resulted with ±12 cm RMS at major part of the country and is degraded to ±20 cm at mountainous region lying along the borders and coastline due to data discontinuity and gravity data insufficiency.

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## 1. INTRODUCTION

Since one of the main purposes of the Geodesy is representation of the earth, measurements should be carried out in a reference system to determine shape and position of the earth. Point positioning on the earth means determining the three dimensional coordinates of the so-called point with respect to a reference system. Point coordinates may be expressed either in Ellipsoidal Geodetic or Geocentric Cartesian Coordinate Systems and consist of three elements ($\varphi$, $\lambda$, h - latitude, longitude and height- or X , Y , Z cartesian coordinates) as shown in Figure 1.

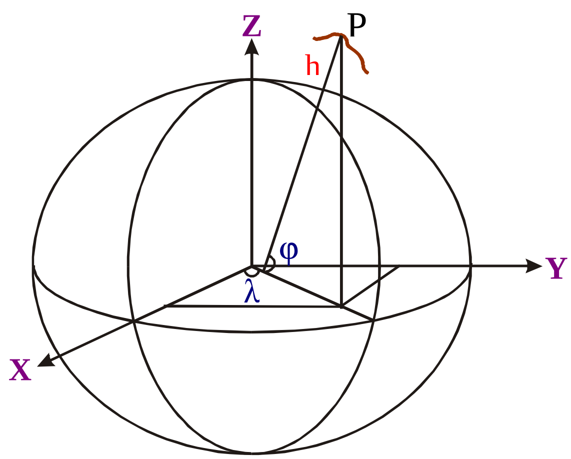



Figure 1: Ellipsoidal Geodetic and Geocentric Cartesian coordinates of a point P.

Conventionally, coordinates were determined in two steps; latitude and longitude ($\varphi$,$\lambda$) were determined by triangulation and astronomic observations, heights by spirit levelling and gravimetric measurements. Global Positioning System (GPS) technology has capability of determining three dimensional coordinates of a point in extremely short time comparing to the conventional geodetic techniques. GPS observations provide latitude, longitude and height of a point referring to a given ellipsoid. This process is realized by observing the baseline between two or more points and express them either in the Geocentric Cartesian coordinate system ($\Delta$X, $\Delta$Y and $\Delta$Z) or in the Local Geodetic Horizon System (LHGS) of north, east and up.

However, instead of ellipsoid height measured by GPS, orthometric height is being utilized in engineering applications. Therefore modeling the difference between gravimetric and GPS/levelling geoids will improve the geoid height determination accuracy at a point observed and, thus, help determination of orthometric heights from GPS observations.

The relationship between orthometric height (H) and ellipsoid height (h) may be expressed as

$$h = N + H \qquad (1)$$

where N is the geoid height, which is the distance between the geoid and the ellipsoid. The problem of transforming Global Positioning System (GPS) derived ellipsoidal height into orthometric height, thus, reduces to determination of the geoid height N.

Basic relationship between geoid height N, ellipsoidal height h and orthometric height H is shown in Figure 2.

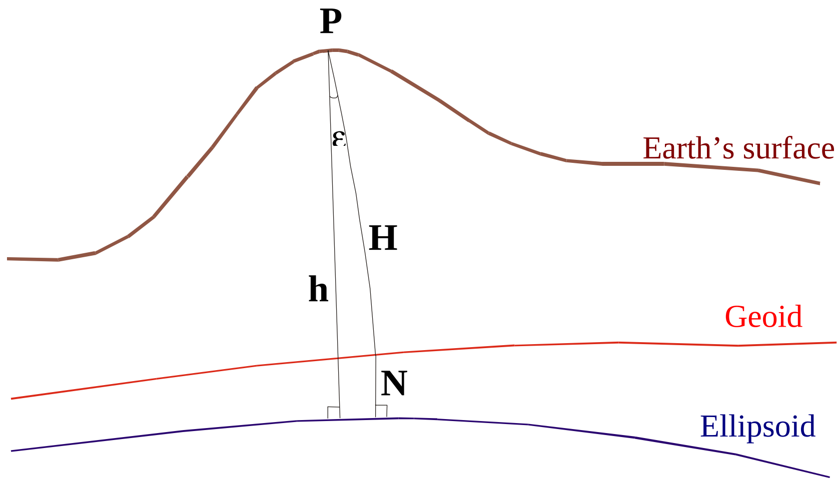



Figure 2 : Relationship between geoid height N, ellipsoidal height h, orthometric height H and vertical deflection ε.

Theoretically, the ellipsoid height and orthometric heights are measured along the normal to the ellipsoid and along the direction of the plumb line (vertical), respectively. These directions at a point do not coincide; therefore, Equation (1) is an approximate relationship. However, even for $\varepsilon = 1''$, the approximation error of the Equation (1) is about $8.10^{-2}$ mm, which is quite insignificant for most surveying applications.

Gravimetric and GPS/leveling geoids of the Turkey that form basis to the processes carried out in the study have been described in sections 2 and 3 respectively. Section 4 is devoted to modeling of the geoid height differences. Section 5 exhibits the evaluation of the result while section 6 reflects the conclusions.

## 2. GRAVIMETRIC GEIOD OF TURKEY

Turkish Geoid–1991 (TG-91) was the first phase of the accurate geoid determination project during last decade (Ayhan 1993). In order to determine the geoid by least squares collocation (LSC), the area covering Turkey was divided into 114 blocks of size $1^o x 1^o$. LSC approximation had been based on upon the tailored geopotential model GPM2-T1, which is complete to degree and order 200 and was developed by tailoring of the model GPM2 to mean free-air anomalies and mean heights of one-degree blocks in Turkey.

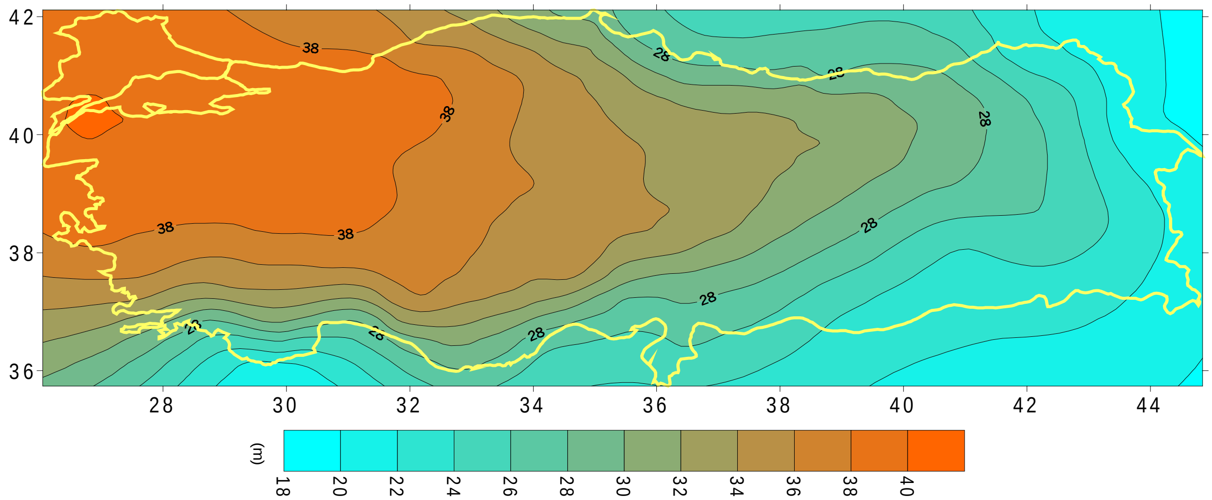



Figure 3: Turkish Geoid 1991 (TG-91)

TG-91 (Figure 3) is referenced to GRS-80 ellipsoid and computed at 3'x 3' grid nodes within the region $36^\circ \leq \Phi \leq 42^\circ$ and $26^\circ \leq \lambda \leq 45^\circ$. Evaluation of the TG-91 had been carried out at gravity points within 14 $1^0 x 1^0$ blocks, which are chosen to represent flat, moderate, and hilly topography. The prediction points had been chosen so that the observations were available but they had not been used in geoid computation. Statistics related to the differences between observed and predicted values have shown that mean and RMS values were larger in hilly area comparing to the flat blocks. Mean and RMS values for differences between observed and predicted values are –0.25 mgal and ±3.01 mgal respectively.

## 3. GPS / LEVELING GEOID HEIGHTS

### 3.1 Determination of Orthometric Heights

Ellipsoidal height is a pure geometrical value while orthometric height has a physical meaning and depends on the gravity field of the earth. Orthometric height of a point P can be computed with the Equation (2):

$$H_p^{ort} = \frac{C_p}{\overline{g}_p} \tag{2}$$

where, $\overline{g}_p$ is the mean value of the gravity along the plumb line and $C_p$ stands for the Geopotential Number of the station (P) and is determined by Equation (3).

$$C_p = C_0 + \sum_{i=1}^{k} \Delta n_i g_i \tag{3}$$

where, $C_0$ is the geopotential number of a benchmark, $\Delta n_i$ is the measured height difference between the points, $g_i$ is average gravity value in each piece of line, k is the number of additional points on the leveling line.

In order to compute the geopotential number, gravity values on the points are obtained by interpolating the observations of the Turkish National Gravity File (TNGF) which is composed in 1994 and consists of the gravity observations all over the country referred to the modified International Gravity Standardization Network 1971 (IGSN 71). The RMS error of the predicted gravity value is less than $\pm 3$ mgal.

GPS stations are connected to at least one vertical control point (benchmark) with second order leveling standards $8\sqrt{S}$ mm (S is distance between points in km). Beforehand, the vertical control point, that will be as used starting benchmark, had been checked with another benchmark for the sake of stability of the point. For this reason, second order leveling was achieved between two benchmarks and height differences were checked whether the difference between ground truth and observed $\Delta n$ was within limits of $10\sqrt{S}$ mm. On the leveling line connecting the benchmark to the GPS point, additional points were constructed at every 1-2 km distance in order to control the height difference. Latitude and longitude of all the points are interpolated on the 1/25000 scale topographic map. After all, the orthometric height difference between the benchmark and the GPS point is observed, and thus, the orthometric height of the point is determined.

### 3.2 Determination of Ellipsoidal Heights of GPS Points

General Command of Mapping (GCM) has conducted a series of countrywide GPS campaigns both individually or in coordination with national and international organizations since 1989. Meanwhile, Turkish National Fundamental GPS Network (TNFGN) had been established between the years 1997-1999 and consists of approximately 600 points distributed all over the country homogeneously (Figure 4).

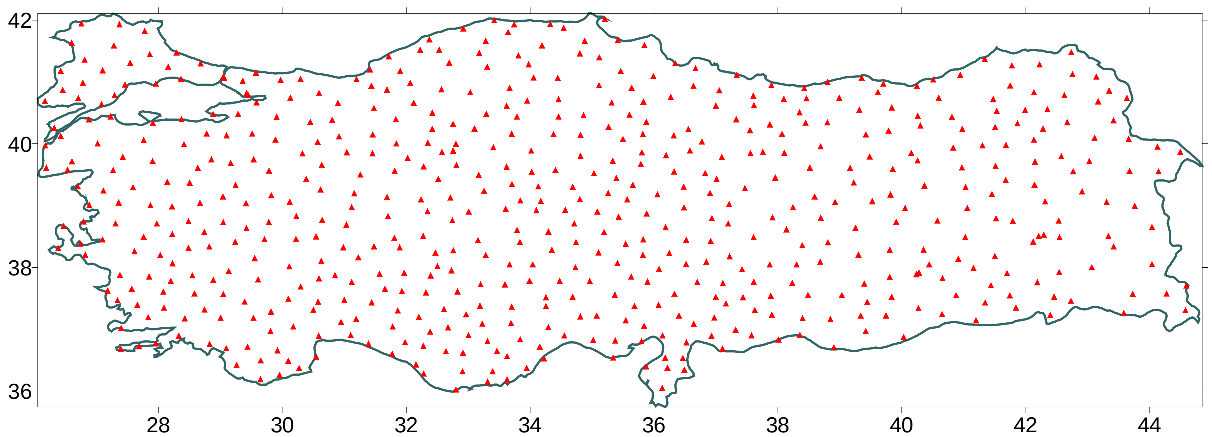



Figure 4 : Turkish National Fundamental GPS Network (TNFGN).

Coordinates of approximately 600 stations that are distributed all over Turkey have been determined in International Terrestrial Reference System 96 (ITRF96). These coordinates consist of (X,Y,Z) Cartesian geocentric coordinates and latitude ($\varphi$), longitude ($\lambda$) and ellipsoidal height (h) on Geodetic Reference System 80 (GRS80) ellipsoid. Thus, the ellipsoidal heights of GPS points have been obtained in ITRF96.

Finally, according to Equation (1), GPS/leveling geoid height ($N_{GPS}$) of a point is determined by subtracting orthometric height ($H^{ort}$) from ellipsoid height (h).

## 4. MODELING THE GEOID HEIGHT DIFFERENCES

### 4.1 Data Preparation

Gravimetric geoid height $N_{GRV}$ predictions at the 187 GPS points were obtained by an interpolation program called GEOIP, which is a Fortran77 code part of GRAVSOFT package (Tscherning et al. 1994). On the other hand, GPS/leveling geoid height $N_{GPS}$ is the result of the difference between the ellipsoidal height h obtained by GPS observations and the orthometric height $H^{ort}$ obtained by spirit leveling and gravity observations. Thus, original data set, ΔN, has been formed according to the Equation (6).

$$\Delta N = N_{GRV} - N_{GPS} \tag{6}$$

In order to eliminate suspicious data, two-phase filtering has been applied to the data set. In the first phase, for statistical reasons, only magnitude of the observation has been taken into the consideration and 3 points are dismissed. In the second phase, an interpretation filtering which depends on neighborhood harmony has been conducted and 5 more points are eliminated. Final data set, comprising of 179 data points, has been formed and the contour map of the data set is shown in Figure 6.

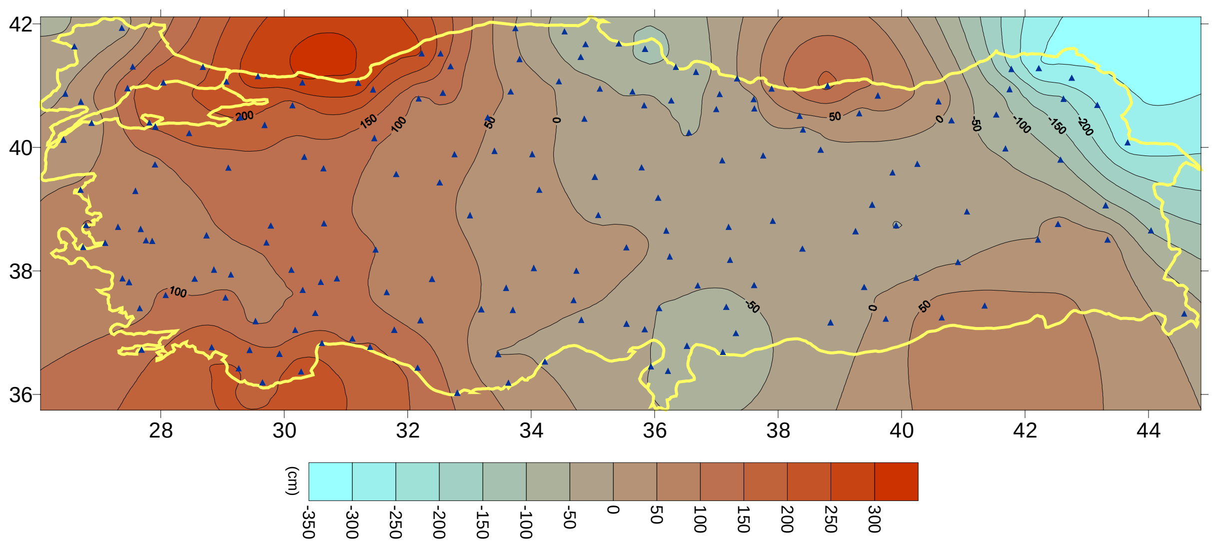



Figure 6 : Geoid height difference data set comprising of 179 points.

### 4.2 Least Squares Collocation

#### 4.2.1 Trend Removal

An important rule that has to be obeyed is that the data has to be centered before the collocation (Moritz 1980). In other words, trend has to be removed from the raw data such that mean of the data would be equal to zero. This trend removal process can be accomplished by applying various trend models to the raw data such as mean removal, first order polynomial fit, second order polynomial fit, etc.

Among four different data detrending models, the model given by Equation 7 was determined as the best trend-removing model (Yildirim 2000)

$$t_i = a_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 y_i^2 + a_5 x_i y_i + a_6 x_i^3 + a_7 y_i^3 + a_8 x_i^2 y_i + a_9 x_i y_i^2 \qquad (7)$$

where $a_i$ denotes the unknown coefficients of the model, $t_i$ stands for the trend part of the geoid height differences, $x_i$ and $y_i$ represent the spherical distance of the i[th] data point with respect to the origin which is taken as $(x_o, y_o) = (36^o, 26^o)$. After calculating the trend component of the data point, it is subtracted from the raw data value to obtain the detrended data.

### 4.2.2 Forming Covariance Matrices

Determination of the covariance function model and its parameters is a prerequisite for composition of the covariance matrices. In this study, covariance function given below has been used (Marti 1993):

$$C_s(r) = C_o (1 + r^2 / D^2)^{-1/2} \qquad (8)$$

where $C_o$ is signal variance and D is the characteristic distance.

Covariance matrices $C_s$ and $C_{s_p s}$ are formed using the same covariance function parameters, $C_o$ and D, that are given in Yildirim 2000.

### 4.2.3 Prediction

After detrending the raw data and forming the covariance matrices, signal prediction has been performed by the Wiener-Kolmogorov formula (Vanicek and Krakiwsky 1982) which is given as:

$$\hat{s}_p = -C_{s_p s}(C_s + C_v)^{-1} l^o \qquad (9)$$

where $\hat{s}_p$ is predicted signal, $C_{s_p s}$ is the cross-covariance matrix between the predicted and observed signal, $C_s$ is covariance matrix of the signal, $C_v$ is covariance matrix of the noise and $l^o$ is vector of observations.

In order to make error assessment, error covariance matrix $C_{\hat{s}}$ of the estimated signal is computed by the given as (Nakiboglu 1997):

$$C_{\hat{s}} = C_s (C_s + C_v)^{-1} C_s \qquad (10)$$

Final process of LSC was adding the removed trend to the computed signal. Thus, geoid height differences were obtained at the 3’x3’ grid corners of the area of interest.

Differences between the original data and data predicted by collocation are named as residuals and they are presented in Figure 8.

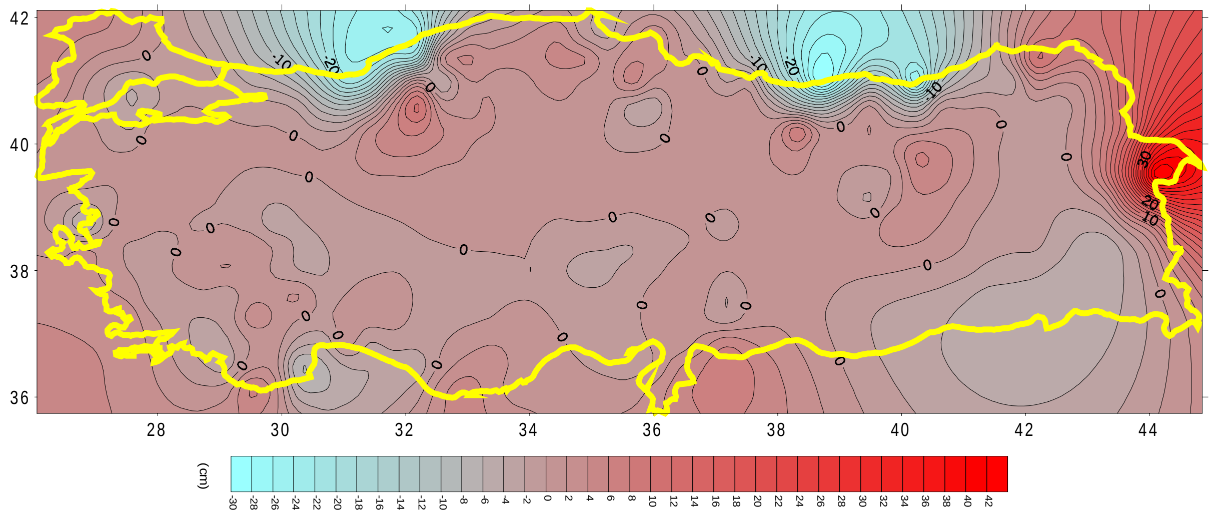



Figure 8 : Residuals obtained from the collocation solution.

## 5. EVALUATION OF THE RESULT

### 5.1 Results

In order to determine orthometric height using GPS observations, two 3'x 3'grid data files composed of geoid height difference data and TG-91 grid data are arranged. A Fortran77 code has been prepared to get the orthometric height by using the data files. Necessary input data are the latitude, longitude and the ellipsoidal height of a point. Output data comprises of latitude, longitude and the orthometric height of the given point. A flowchart illustration about this process is given in Figure 10:

Input GPS coordinates :
latitude ( $\varphi$ ), longitude( $\lambda$ ) and ellipsoidal height (h)

↓

Gravimetric geoid height determination by
interpolating TG-91 grid file ($N_{GRV}$)

↓

Geoid height difference determination by interpolating
collocation grid file ( $\delta N = \text{trend} + \text{signal}$ )

↓

Determination of orthometric height H by
$H = h - (N_{GRV} - \delta N)$

Figure 10 : Flowchart of the orthometric height derivation process

Additionally, a user- friendly program has been prepared by using the Graphical User Interface (GUI) tool kit of MATLAB 5.3 software. This program can be operated on personal computers on which MATLAB software has been installed. Program is capable of reading input data from a formatted file and writing results to a designated file. Optionally, input data could be introduced to the program manually and output could be seen on the display.

Standard deviation of the orthometric height, H, can be given as

$$\sigma_H = \pm\sqrt{\sigma^2_{N_{GRV}} + \sigma^2_h + \sigma^2_{\delta N}} \tag{11}$$

where $\sigma_{N_{GRV}}$ is standard deviation of the gravimetric geoid height, $\sigma_h$ is standard deviation of the ellipsoidal height measured by GPS, $\sigma_{\delta N}$ is standard deviation of the predicted data by collocation.

Taking the mean standard deviation of the gravimetric geoid height as $\pm 14\,\mathrm{cm}$, the ellipsoidal height as $\pm 2\,\mathrm{cm}$ and the predicted data as $\pm 14\,\mathrm{cm}$, mean accuracy of the orthometric height along the borders comes out to be $\pm 20\,\mathrm{cm}$ while it comes down to $\pm 12\,\mathrm{cm}$ in central part of the country taking $\sigma_h = \pm 2\,\mathrm{cm}$, $\sigma_{N_{GRV}} = \pm 9\,\mathrm{cm}$ and $\sigma_{\delta N} = \pm 8\,\mathrm{cm}$.

## 5.2 Test of the System Performance

Performance of the model is tested with three independent data sets comprising 33 points which are observed in the vicinity given in Figure 11.

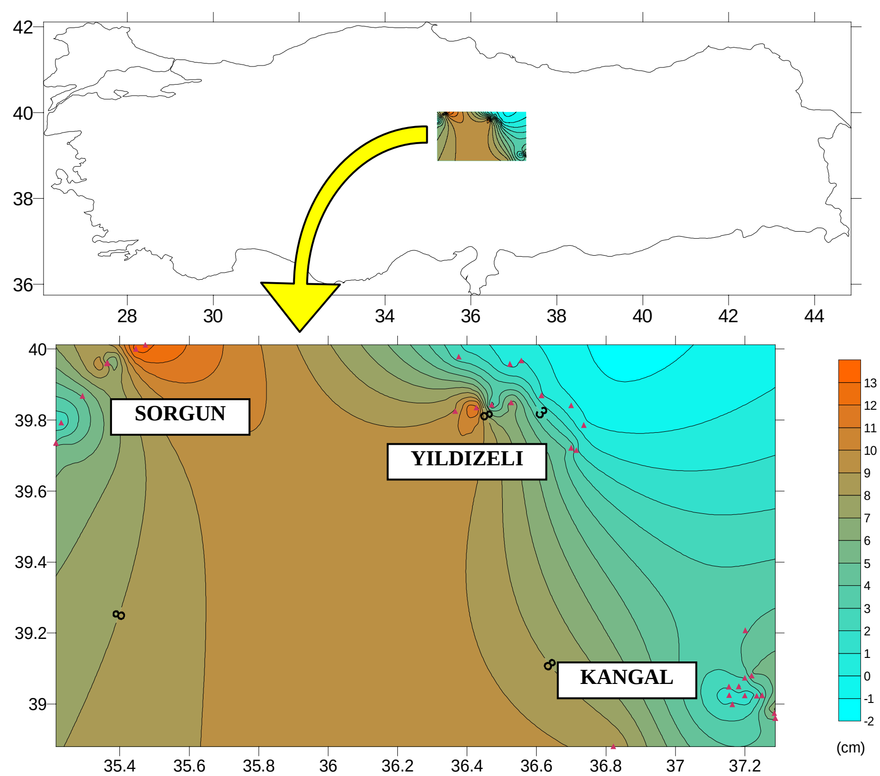
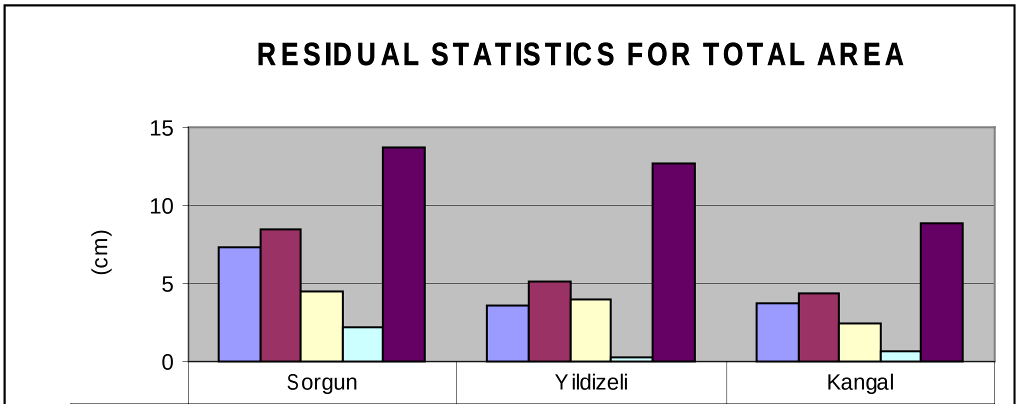



Figure 11 : Location of the test points.




| | Sorgun | Yildizeli | Kangal |
|---|---|---|---|
| Mean | 7.31 | 3.53 | 3.68 |
| RMS | 8.44 | 5.19 | 4.34 |
| Std.Dev. | 4.50 | 3.97 | 2.41 |
| Min. | 2.23 | 0.20 | 0.70 |
| Max. | 13.70 | 12.70 | 8.80 |

Figure 12 : Graph and the table of residuals obtained after test observations.

Residuals obtained for all three test sites are presented in a graph given in Figure 12. It is seen that residuals are well below the presumed error estimate. The main reason of these fairly good test results is thought be relavant to the density of the GPS/leveling points in this area. Another important fact that has improved the accuracy of the results is the accuracy of the gravimetric geoid TG-91 in the vicinity.

It is quite hard to expect same optimistic results along the border-lines since there are condiderable big gaps between control points and there exist relatively inaccurate gravimetric geoid heights due to the lack of sufficient gravity data during computation.

## 6. CONCLUSIONS

It can be confidently expressed that modeling the difference between gravimetric and the GPS/leveling geoid heights has been very successful to some extent and determining the orthometric height of a point, on which GPS observation had been conducted, has become possible with certain accuracy.

Accuracy achieved is thought be quite sufficient for many of the engineering applications comparing to the alternative method spirit leveling, which is a time and money consuming labor.

Computation of a new gravimetric geoid model for Turkey, using surrounding sea and land gravity data, will dramatically increase the accuracy of the orthometric height values that are derived from GPS observations.

Testing the performance of the collocation solution obtained from this study at various sites, each having different topographical structure, will contribute to the accuracy of the solution.

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## BIOGRAPHICAL NOTES

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He has graduated from Istanbul Technical University, Civil Eng. Faculty Dept. Of Geodesy and Photogrammetry in 1985. Following his start as Research Assistant in 1986 and having completed his PhD in 1993 on GIS and after becoming Asst. Professor Dr. in 1996 respectively, He has been working as Assoc. Prof. Dr. since 1997. He had worked in the same depatment as Research Assistant till 1996 and now he is still working in the same place as Associated Professor.

### Abdullah YILDIRIM

After graduating from Army Academy in 1983, he attended to Army Aviation School and began to serve as pilot at Aviation Group of General Command of Mapping (GCM) in 1985. Following the AP3 Navigation Course of ITC/Netherlands in 1988, he also began to fly as Navigator Pilot. He graduated in 1997 as survey engineer in Geodesy & Photogrammetry and obtained his MSc degree in 2000. The subject of his MSc thesis was "Modeling the Difference Between Gravimetric and GPS/Leveling Geoids". In this study, he concentrated on improving the accuracy of the local gravimetric geoid in order get accurate orthometric heights from GPS obsevations. He has been lecturing in Istanbul Technical University for PhD degree. He is now working at mission planning section of GCM head-quarters.