Comparative Analysis of Geodetic Distance Computational Methods, Using the Normal Probability Statistical Plot

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SUMMARY (Abstract)

There are various methods that can be used by Geodesist in carrying out computation of geodetic distance using geodetic coordinates (latitude and longitude) on the ellipsoid. Some of these methods require iterations while others need no iterations. Iterative methods are complex while non – iterative are simple and faster in performing computations. The research looks into three different geodetic computation methods which are: Bowring, Power series and Puissant. The geodetic coordinates were plotted and triangulation networks were formed. The adjoining distances sections were divided into three categories namely; short, medium and long baselines. The validity of each method with regards to the distances was used in the comparisons. One way analysis of variance (ANOVA) was performed on each method with respect to the baselines. The p – values of each of the methods were plotted on the normal probability graph for a comparative analyses. Based on the findings described in this research, conclusion was made on an appropriate method that is best for a particular baselines computation. The three methods of geodetic computation considered in this research work were actually good for computation of distances but each of the method was valid for a particular range of baselines. Bowring method is best used for long baselines computation. The accuracy of Bowring method becomes better as the baselines increases. Power series method is best used for short and long distances. Puissant method was valid for both short and medium baselines.

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1.0 INTRODUCTION

The method of geodetic computations are completely different from planar computation method. Geodetic computations of distances and positions are complex and rigorous (Christopher 2006; Douglas et.al 2003). It is compulsory to take into considerations the curvature of the earth because; large portion of the surface of the earth is covered. The earth radius, semi major axis and flattening are crucial parameters to be considered during computations (DMA 1984; Deakin et.al 2010). The complexity of geodetic computations can be traced to series mathematical equations and iterations performed. The geodetic computation makes use of geodetic coordinates; latitudes and longitudes. In addition measurement of gravity field is considered because geodesy instruments use gravity as reference (Eteje et.al 2019; GDA 2014). The measurement of long baselines and determination of positions require more accurate observing instruments than in surveying. Although, surveyors and geodesists use the same instruments most times (John 1996; Wolfgang 2001).

Geodetic surveys are linked to reference frames (networks) established by global geodesy; these surveys the global parameters for the figure of the earth and its gravity field. On the other hand, the results of geodetic surveys may contribute to the work of the global geodesist. Plane surveys, in turn, are generally referenced to control points established by geodetic surveys. The measurement and data evaluation methods used in national geodetic surveys are often similar to those used in global geodetic work (Wolfgang 2001).
Ellipsoid of rotation is considered as the best approximation to the size and shape of the earth, it is used as the surface upon which to perform terrestrial geodetic computations. An ellipsoid of revolution is specially defined by two dimensions specifically. Geodesists use the semi major axis and flattening conventionally. The size is denoted by the radius at the equator – the semi major axis and letter, a, is used to representing it. The ellipsoid shape is denoted by flattening, f, which indicates how closely an ellipsoid approaches a spherical shape. The difference between the ellipsoid of revolution representing the earth and sphere is very small.

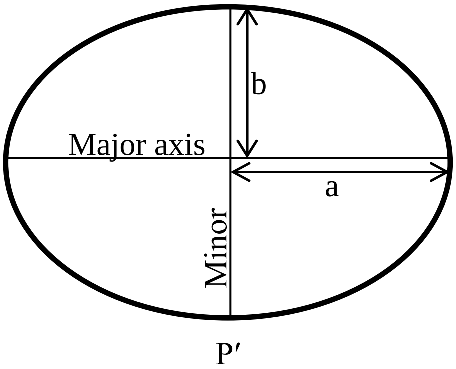

Figure 1.0: Ellipsoid of rotation

a = semi major axis

b = semi minor axis

f = flattening = (a-b/a)

PP′ = axis of revolution of the earth ellipsoid

A Reference System is a set of prescriptions and conventions together with the modelling required defining at any time a triad of coordinate axes. Geodetic reference systems provide numerical values for the parameters of a geodetic earth model. The system permits the spatial referencing of all land data to identifiable positions on the Earth's surface (James 1997; Joenil 2004). A geodetic reference system provides not only an accurate and efficient means for positioning data, but it also provides a uniform, effective language for interpreting and disseminating land information. These Reference Systems are established in order to model geodetic observations as a function of unknown parameters of interest.
The orientation of geodetic systems with respect to the earth is described by the "Geodetic Datum" (National & University Library). Datum is defined as numerical or geometrical quantity or set of those quantities which are used as a reference or base for other quantities (Richard 1991). There are two types of datum that are considered in geodesy, namely: horizontal and vertical datum. Horizontal datum is the basis for computations of horizontal control surveys in which the curvature of the earth is considered and a vertical datum is vertical controls to which elevations are referred (DMA 1983).

---- SIGNIFICANCE OF STUDY

The purpose of this study was to perform analysis and determine validity of various methods mentioned earlier on using existing geodetic control network data. The computed data from various methods were compared with the existing geodetic data ((Krakiwsky et.al 1974; Jure 2008). Various methods had their short comings, but deductions were made for methods that were valid for the determination of length of geodetic distances using computational methods (John 1996; Martin 2019). The research work also reveals the importance of using these computational methods for obtaining geodetic data, as its saves time and cost (Ozge et.al 2016). Its primary advantage is that, it offers the ability to rapidly and non-intrusively obtained adequate geodetic control data from the existing data.

---- AIM: Computation of distances, forward azimuth, and back azimuth of triangulation network from longitudes and latitudes of points using the methods of: Bowring, Power series and Puissant.

Objectives:

(i) Plot triangulation networks from geodetic coordinates using MATLAB application.

(ii) Distances, forward and back azimuth computation with inverse solution.

(iii) Computation of distances, back azimuth, latitudes and longitudes with direct solution (iv) Analysis of the various models using statistical methods.

---- AREAS OF STUDY

Bauchi state is located between latitudes 9° 3' and 12° 3' north and longitudes 8° 50' and 11° east. The state is bordered by seven states, Kano and Jigawa to the north, Taraba and Plateau to the south, Gombe and Yobe to the east and Kaduna to the west. **Kano state** is located in the north-western part of the country. It is situated between latitudes 11° 25' N to 12°47' N and longitude 8° 22' E to 8°39'E east and 472m above sea level. **Plateau** is tropical highland, near the centre of Nigeria. The plateau covers an area of about 7,800 sq. km (about 3,000 sq. mi) and lies at a general altitude of about 1,300 m (about 4,300 ft.) (Microsoft Encarta 2009)

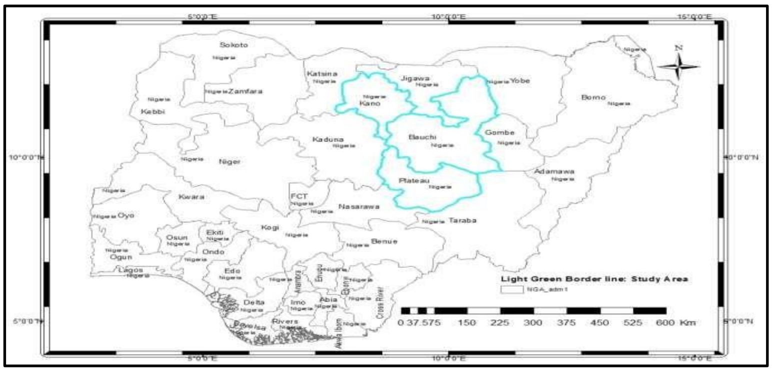

Figure 1.0: Map of Nigeria showing area of study in light green border line.

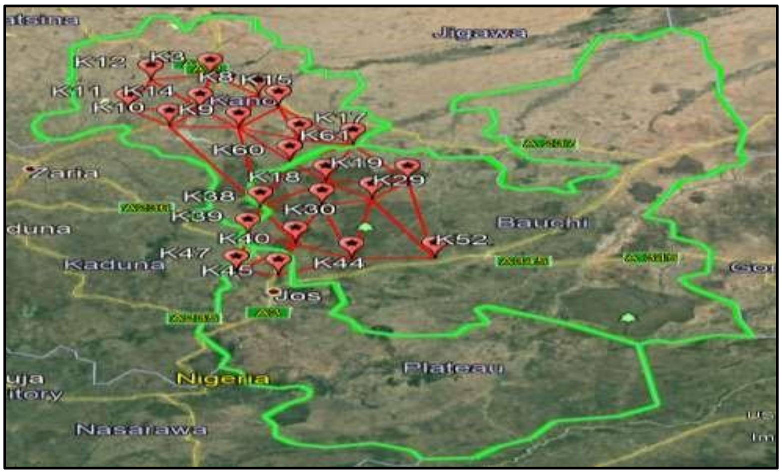

Figure 1.1: Google earth image of triangulation network of geodetic points on ground. Green line depicts area of study, red line depicts the triangulation network and red place mark shows the geodetic positions on ground.

2.0 LITERATURE REVIEW

---- DIRECT GEODETIC PROBLEM AND INDIRECT GEODETIC PROBLEM

The direct geodetic problem is that kind of problem that determines the second coordinates and back azimuth, when the coordinates of first point, the distance from the first point to second point and forward azimuth is known on the rotation ellipsoid. The direct problem can be expressed in a functional way as follows:

Direct problem: $\phi_2 = f_1 (\phi_1, \lambda_1, \alpha_{12}, s)$

$$\lambda_2 = f_3 (\phi_1, \lambda_1, \alpha_{12}, s)$$

$$\alpha_{21} = f_3 (\phi_1, \lambda_1, \alpha_{12}, s)$$

The indirect problem is also referred to as the reverse or inverse problem. The indirect problem can be solved from given coordinates of the two points, the problem aim at determining the forward azimuth, back azimuth and distance between the two known points on the ellipsoid. The direct problem and inverse problem can be expressed in a functional way as follows:

Inverse problem: $s = f_4 (\phi_1, \lambda_1, \phi_2, \lambda_2)$

$$\alpha_{12} = f_5 (\phi_1, \lambda_1, \phi_2, \lambda_2)$$

$$\alpha_{21} = f_6 (\phi_1, \lambda_1, \phi_2, \lambda_2)$$

---- GEDOETIC COMPUTATION METHODS

---- BOWRING FORMULAR

Bowring method of geodetic computation was derived in 1981 by Bowring. The equations were both for the direct and inverse problems. This method can be used to solving problem for the geodesic lines up to 150km in length. The process of computing direct and inverse problem using the Bowring method is non-iterative (Rapp 1991). The accuracy indicates 1 or 2mm for the direct or inverse solution for lines on the order of 120km in length. For lines that are up to 150km in length the error in an inversed distance increased to 3 or 4 mm while for lines up to 100 km has the azimuth error in the order of 0.001 second. (Rapp 1991).

---- PUISSANT FORMULA

Puissant formula is named after the French mathematician who is credited to have developed the formula. The derivation of this formula is based on a spherical approximation; as a result, the formula is considered to be correct to 1 ppm at 100km, beyond 100km it break down rapidly to 40 ppm at 250km when latitude is 60◦ (Bomford 1971). Therefore, Puissant formula is a short line formula. The equations were initially derived by Puissant in the 18th century. They have been extended and used by a number of different geodetic organizations for position computations. The equations were not derived with great rigor and are not used mostly for computation of lines greater than 100km in length (Rapp 1991). The necessary equations can be derived for the direct problem by considering a sphere of radius N_1, tangent along the parallel through the first point. The sphere will be approximately coincident with the second point when equations are derived for short distances. We assume that the azimuth and distance are the same on the sphere and on the ellipsoid.

---- POWER SERIES METHOD

According to Rapp (1991), it is assumed that, a curve on the ellipsoid can be expressed as a function of s in order to provide a solution to direct problem in geodetic positioning. (Bagratuni 1967) shows that the accuracy of this formula can be used up to 130km. Although,
Grushinsky (1969) indicates that formulas like this are valid up to 600 – 800km in lengths. Solution of the inverse problem using series expansions is not direct. The problem is solved in an iterative process.

3.0 METHODOLOGY 3.1 DATA ACQUISITION

Geodetic control stations have been established across Nigeria many years ago. The K chain geodetic coordinates were used for the research work. Minna Datum was the geodetic datum used in this work. Minna datum is referenced to Clarke 1880 (RGS) ellipsoid. The datum has the following parameters: Semi major axis, a = 6378249.145m; Flattening, f = 1/293.465. The geodetic data were shown in the table 3.1 below.

Table 3.1: Geodetic data

POINT NO.	POINT ID	LATITUDE	LONGITUDE
1	K10	11.50628047	8.328956664
2	K11	11.65588272	8.127462386
3	K12	11.94303061	8.219130125
4	K14	11.67079781	8.463655531
5	K15	11.70690764	8.826188003
6	K17	11.35096831	9.187165356
7	K18	10.99870883	9.077301822
8	K19	11.01187592	9.452680667
POINT NO.	**POINT ID**	**LATITUDE**	**LONGITUDE**
9	K29	10.83795144	9.296884339
10	K3	12.00217167	8.493034392
11	K30	10.76705367	9.069939825
12	K38	10.73402739	8.786830567
13	K39	10.47508492	8.740865108
14	K40	10.40356489	8.963393742
15	K44	10.2624955	9.22617815
16	K45	10.09523161	8.896563478
17	K47	10.11895461	8.701368939

18	K52	10.28318944	9.604576872
19	K60	11.18385747	8.903194631
20	K61	11.39328067	8.937091061
21	K8	11.81307653	8.729067569
22	K9	11.49398153	8.651509436

3.2 PLOTTING OF GEODETIC DATA

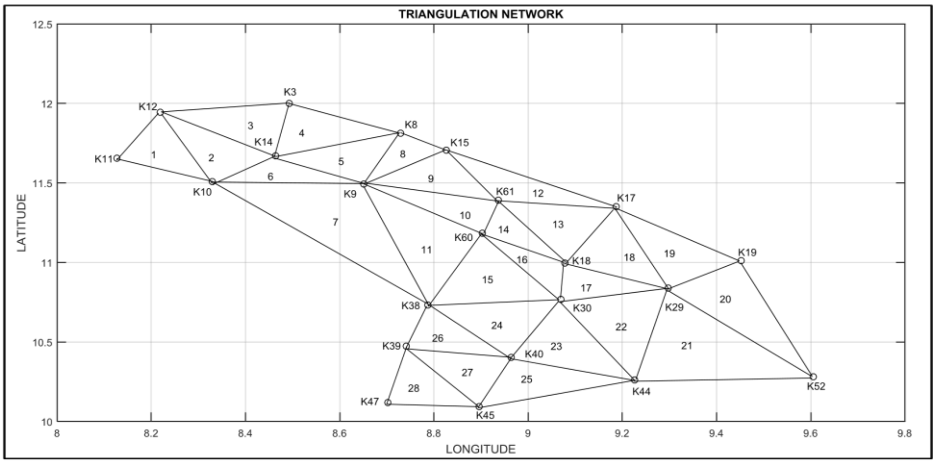

Figure 3.0: Triangulation network

---- BOWRING METHOD

This formula for computing the direct and inverse problems of the geodesic for lines that are about 150km in length was derived in 1981 by Bowring. The process of providing solutions to the direct and inverse problems is non- iterative. The equations 3.1 to 3.6 were used for solution in both inverse and direct problems using Bowring method are:

$A = (1 + e'^2 cos^4\, \phi_1)^{\frac{1}{2}}$ Equation 3.1

$B = (1 + e'^2 cos^2 \phi_1)^{\frac{1}{2}}$ Equation 3.2

$C = (1 + e'^2)^{\frac{1}{2}}$ Equation 3.3

$\omega = \frac{A(\lambda_2 - \lambda_1)}{2}$ Equation 3.4

$\Delta\phi = \phi_2 - \phi_1$ Equation 3.5

$\Delta\lambda = \lambda_2 - \lambda_1$ Equation 3.6

Inverse Problem

Solution to the inverse problem using Bowring method is simply carried out by using the parameters given in section 3.3.0 and table 3.1. This helped us in computing equations 3.1 to 3.6. Thereafter, we proceeded to computing equations 3.7 to 3.16.

$D = \frac{\Delta\phi}{2B}\,[1 + \frac{3e'^2}{4B^2}\,\Delta\phi \sin(2\phi_1 + \frac{2}{3}\Delta\phi)]$ Equation 3.7

$E = sinDcos\omega$ Equation 3.8

$F = \frac{1}{A} sin\omega(Bcos\phi_1 cosD - sin\phi_1 sinD)$ Equation 3.9

$G = tan^{-1}\left(\frac{F}{E}\right)$ Equation 3.10

$h = 0$ Equation 3.11

$sin\frac{\sigma}{2} = (E^2 + F^2)^{\frac{1}{2}}$ $therefore,$ $\sigma = asin2(E^2 + F^2)^{\frac{1}{2}}$ Equation 3.12

$H = tan^{-1}\left[\frac{1}{A}(sin\phi_m + Bcos\phi_1 tanD)tan\omega\right]$ Equation 3.13

$Forward\ azimuth,\ \alpha_{12} = G - h$ Equation 3.14

$Back\ azimuth,\ \alpha_{21} = G + H \pm 180°$ Equation 3.15

$Distance, s = aC\sigma/B^2$ Equation 3.16

Where σ is the meridian distance.

Direct problem

Solution to the direct problem using Bowring method is simply carried out by using the parameters given in section 3.3.0 and table 3.1 This helped us to compute equations 3.1 to 3.6.
Thereafter, we proceeded to computing equations 3.17 to 3.21.
The equations for solving direct problem using Bowring method are:

$D = \frac{1}{2} sin^{-1}[sin\sigma\left(cos\alpha_{12} - \frac{1}{A} sin\phi_1 sin\alpha_{12} tan\omega\right)]$ Equation 3.17

$\sigma = \frac{SB^2}{ac}$ Equation 3.18

$Longitude\ of\ the\ second\ point,\ \lambda_2 = \lambda_1 + tan^{-1}\left(\frac{Atan\sigma sin\alpha_{12}}{Bcos\phi_1 - tan\sigma sin\phi_1 cos\alpha_{12}}\right)$ Equation 3.19

$Latitude\ of\ the\ second\ point,\ \phi_2 = \phi_1 + 2D\left[B - \frac{3e'^2}{2} Dsin\left(2\phi_1 + \frac{4}{3}BD\right)\right]$ Equation 3.20

$Back\ azimuth,\ \alpha_{12} = tan^{-1}\left[\frac{-Bsin\alpha_{12}}{cos\sigma(tan\sigma tan\phi_1 - Bcos\alpha_{12}}\right]$ Equation 3.21

---- POWER SERIES FORMULA

We started by providing solution to inverse problem by solving the equations 3.22 to 3.24 below:

$V = \sqrt{1 + e'^2 cos^2\phi}$ Equation 3.22

$N = \frac{a}{(1 - e^2\ Sin\phi_1)^{1/2}}$ Equation 3.23

$M = \frac{a(1 - e^2)}{(1 - e^2\ Sin^2\ \phi)^{3/2}}$ Equation 3.24

Inverse Problem

The solution of the inverse problem under the power series formula is not direct. The solution was provided by using the first terms of the equations 3.25 and 3.27 below in an iterative procedure.

$$\phi_2 - \phi_1 = \frac{V_1{}^3}{c} cos\alpha_{12} s + \Delta_A$$ Equation 3.25

$$\lambda_2 - \lambda_1 = \frac{V_1}{c} \frac{sin\alpha_{12}}{cos\phi_1} s + \Delta_B$$ Equation 3.26

$c = NV \ or \ c = MV^3$ where: c is the radius of curvature at the pole and V is the vertical angle,

$\Delta_A and \ \Delta_B$ are functions of distance, s, forward azimuth $\propto_{12}$, and latitude of first point ϕ_1. The equations 3.25 and 3.26 were solved by assuming that, $\Delta_A and \ \Delta_B$ were known. Therefore, we have equation 3.66.

$$\Delta\phi - \Delta_A = \frac{V_1{}^3}{c} cos\alpha_{12} s$$ Equation 3.27

$$\Delta\lambda - \Delta_B = \frac{V_1}{c} \frac{sin\alpha_{12}}{cos\phi_1} s$$ Equation 3.28

The equation 3.27 and 3.28 were divided and rearranged so that we have:

$$tan \propto_{12} = V_1{}^2 \ cos\phi_1 \ [\frac{\Delta\lambda - \Delta_B}{\Delta\phi - \Delta_A}]$$ Equation 3.29

The distance s, was solved by using equation 3.27

$$s = \frac{c(\Delta\phi - \Delta_A)}{V_1{}^3 \cos \alpha_{12}}$$ Equation 3.30

We have the known values $\Delta\phi$ and $\Delta\lambda$, therefore we solve for the first approximation of forward azimuth using equation 3.31

$$tan \propto_{12}{}^{(1)} = V_1{}^2 \ cos\phi_1 \ [\frac{\Delta\lambda}{\Delta\phi}]$$ Equation 3.31

The first approximation of distance was computed using equation 3.30

$$s^1 = \frac{c\Delta\phi}{V_1{}^3 cos\propto_{12}}$$ Equation 3.32

We computed for the values $\Delta_A and \ \Delta_B$ using equations 3.27 and 3.28, as a result, equations 3.29 and 3.30 were computed. The results were compared with the first approximation of equations 3.31 and 3.32. We continued the process of iteration until the values of forward azimuth and distance failed to change.

Direct Problem

The equations 3.33 to 3.36 below were used for solving direct problems. Given the latitude (ϕ_1), and the longitude (λ_1), of a point, forward azimuth (α_{12}), and distance (s) to another point of unknown latitude (ϕ_2), and the longitude (λ_2), we computed for the back azimuth (α_{21}), latitude, ϕ_2 and longitude , λ_2.

Where, $t, \eta^2, u \ and \ v$ *are:*

$$t = tan\phi$$ Equation 3.33

$\eta^2 = e'^2 cos^2\phi$ Equation 3.34

$u = \frac{Vscos\alpha}{c}$ Equation 3.35

$v = \frac{Vssin\propto}{c}$ Equation 3.36

Where: u and v are the local coordinates, t is a quantity

Equations 3.37 were used for solving the latitude ϕ_2, longitude, λ_2 and back azimuth of the second point:

$$\phi_2 = V^2 \begin{bmatrix} u - \frac{v^2 t}{2} - \frac{3u^2\eta^2 t}{2} - \frac{v^2 u}{6}(1 + 3t^2 + \eta^2 - 9\eta^2 t^2) - \frac{u^3\eta^2(1-t^2)}{2} \\ + \frac{v^4 t}{24}\left(1 + 3t^2 + \eta^2 - 9\eta^2 t^2\right) \\ -\frac{v^2 u^2 t}{12}(4 + 6t^2 - 13\eta^2 - 9\eta^2) \\ + \frac{u^4\eta^2 t}{2} + \frac{v^4 u}{120}(1 + 30t^2 + 45t^4) - \frac{v^2 u^3}{30}(2 + 15t^2 + 15t^4) \end{bmatrix} + \phi_1$$

Equation 3.37

$$\lambda_2 = \begin{bmatrix} v + vut - \frac{v^3 t^2}{3} + \frac{vu^2}{3}(1 + 3t^2 + \eta^2) \\ -\frac{v^3 ut}{3}(1 + 3t^2 + \eta^2) \\ + \frac{vu^3 t}{3}(2 + 3t^2 + \eta^2) + \frac{v^5 t^2}{15}(1 + 3t^2) \\ + \frac{vu^4}{15}(2 + 15t^2 + 15t^4) \\ -\frac{v^3 u^2}{15}(1 + 20t^2 + 30t^4) \end{bmatrix} \div \cos(\phi_1) + \lambda_1$$

Equation 3.38

$$\alpha_{21} = [(\alpha_{12} \pm 180°) + vt + \frac{vu}{2}(1 + 2t^2\eta^2) - \frac{v^3}{6}t\left(1 + 2t^2 + \eta^2\right) + \frac{vu^2}{6}t\left(5 + 6t^2 + \eta^2 - 4\eta^4\right) - \frac{v^3 u}{24}\left(1 + 20t^2 + 24t^4 + 2\eta^2 + 8\eta^2 t^2\right) + \frac{vu^3}{24}\left(5 + 28t^2 + 24t^4 + 6\eta^2 + 8\eta^2 t^2\right) + \frac{v^5}{120}t(1 + 20t^2 + 24t^4) - \frac{v^3 u^2}{120}t(58 + 280t^2 + 240t^4) + \frac{vu^4}{120}t(61 + 180t^2 + 120t^4)].$$

Equation 3.39

---- PUISSANT FORMULA

Solutions to both inverse and direct problems were provided using the equations 3.40 to 3.48 below:

$N = \frac{a}{(1 - e^2 sin^2\phi)^{1/2}}$ Equation 3.40

$M = \frac{a\left(1 - e^2\right)}{(1 - e^2 Sin^2 \phi)^{3/2}}$ Equation 3.41

$$B = \frac{1}{M_1}$$ Equation 3.42

$$C = \frac{tan\phi_1}{2M_1\, N_1}$$ Equation 3.43

$$D = \frac{3e^2 sin\phi_1 cos_1}{2(1-e^2 sin^2\phi_1\,)}$$ Equation 3.44

$$E = \frac{1+3tan^2\phi_1}{6N_1^2}$$ Equation 3.45

$$h = \frac{scos\alpha_{12}}{M_1}$$ Equation 3.46

$$\Delta\alpha = \Delta\lambda sin\phi_m$$ Equation 3.47

$$\delta\phi = \frac{s}{M_1} cos\alpha_{12} - \frac{s^2}{2N_1M_1} sin^2\alpha_{12} tan\phi_1 - \frac{s^3}{6N_1^2M_1} sin^2\alpha_{12}\, cos\alpha_{12}\, (1 + 3tan^2\phi_1)$$

Equation 3.48

Inverse Problem

The solution to indirect problem using Puissant formulas is iterative. Firstly, we computed an approximate value of equation 3.49. At first approximation, the denominator is assumed to be equals to one then, $ssin\alpha_{12}$was computed.

$$ssin\alpha_{12} \;=\; \frac{N_2\Delta\lambda cos\phi_2}{[1-\frac{s^2}{6N_2^2}(1-sin^2\,\alpha_{12}\, sec^2\phi_2)]}$$ Equation 3.49

Also, we proceeded to solving the approximate value $scos\alpha_{12}$ in equation 3.50

$$scos\alpha_{12} = \frac{1}{B}\,[\Delta\phi + s^2 sin^2\alpha_{12}\, C + hEs^2 sin^2\alpha_{12} + (\delta\phi)^2 D]$$ Equation 3.50

The unknowns at the right hand were set to zero in order to obtain approximate value of$scos\alpha_{12}$. The approximate values of forward azimuth, α_{12} and distance, s were computed using equations 3.51 and 3.52.

$$Forward\ azimuth,\ \alpha_{12} = tan^{-1}(\frac{ssin\alpha_{12}}{scos\alpha_{12}})$$ Equation 3.51

$$Distance, s = \;((ssin\alpha_{12})^2 + (scos\alpha_{12})^2)^{\frac{1}{2}}$$ Equation 3.52

We again carried out computations of equations 3.49 and 3.50 using the computed values from equations 3.51 and 3.52. The iteration process continues until the values obtained of forward azimuths and distances do not changed.

Direct Problem

Solutions to direct problem in Puissant formula were done following the same procedures used in computing the solutions to the inverse problem. The equations were different but the number of iterations was the same.

Equations of computing direct problem in Puissant formulas:

$$\Delta\phi = scos\alpha_{12}\, B - s^2 sin^2\alpha_{12}(hs^2 sin^2\alpha_{12} E - (\delta\phi)^2 D$$ Equation 3.53

$$\Delta\lambda = \frac{s}{N_2} sin\alpha_{12} sec\phi_2 [1 - \frac{s^2}{6N_2^2}(1 - sin^2\alpha_{12} sec^2\phi_2)]$$ Equation 3.54

$$Latitude\ of\ the\ second\ point, \phi_2 = \phi_1 + \Delta\phi \quad \text{Equation 3.55}$$

$$Longitude\ of\ the\ second\ point, \lambda_2 = \lambda_1 + \Delta\lambda \quad \text{Equation 3.56}$$

$$Back\ azimuth, \alpha_{21} = \alpha_{12} + \Delta\alpha \pm 180° \quad \text{Equation 3.57}$$

4.0 RESULTS AND ANALYSIS

---- STATISTICAL ANALYSIS OF RESULTS

Tables 4.1a to 4.3c show the statistics analysis results of the baselines from the geodetic methods. The one way Analysis of Variance (ANOVA) test was to prove if all the sample means were equal or at least one is different. H_0 is the null hypothesis which implies that all sample means are equal and H_1 is the alternative hypothesis, it implies that at least one sample mean is significantly different from others. The p – value and F – critical were measures used for accepting the hypothesis results. The alpha which was the significance level was equal to 0.05. If p – value was less than the alpha level, hypothesis will be rejected but if greater it will be acceptable. Also, if F – statistics was less than F – critical, result of the hypothesis will be accepted. Figures 4.1 to 4.3 show further in graphical expressions the analysis of various methods with respect to types of baselines.

---- ANALYSIS OF BASELINES (DISTANCES) IN KILOMETRES

The baselines of the triangulation network were divided into three (3) categories namely: short baselines (below 40km), medium baselines (between 40km and 100km) and long baselines (above 100km). A random selection was made for ten (10) baselines from all the categories distances. The tables 4.1a to 4.3c are the categories of baselines. Analysis of variance (ANOVA) test was performed to prove the significance of each method. The p – values of each baselines were expressed graphically for the three methods.

Table 4.1a: Analysis of Variance (ANOVA) of Short Distances with Bowring

	Source of the variability.	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)
Short Distances (km)	variability *between* groups	0.0072255	1	0.0072255	0.00016145	0.99
	variability *within* groups	805.5507	18	44.7528		
	Total variability	805.5579	19			

Table 4.1b: Analysis of Variance (ANOVA) of Short Distances with Power series method

	Source of the variability.	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)

Short Distances (km)	variability *between* groups	0.0004295	1	0.0004295	9.9184e-06	0.99752
	variability *within* groups	779.4745	18	43.3041		
	Total variability	779.475	19			

Table 4.1c: Analysis of Variance (ANOVA) of Short Distances with Puissant method

	Source of the variability.	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)
Short Distances (km)	variability *between* groups	0.0019385	1	0.0019385	4.4763e-05	0.99474
	variability *within* groups	779.498	18	43.3054		
	Total variability	779.5	19			

Table 4.2a: Analysis of Variance (ANOVA) of Medium Distances with Bowring method

	Source of the variability.	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)
Medium Distances (km)	variability *between* groups	0.024995	1	0.024995	0.00012326	0.99126
	variability *within* groups	3649.8743	18	202.7708		
	Total variability	3649.8993	19			

Table 4.2b: Analysis of Variance (ANOVA) of Medium Distances with Power series method

	Source of the variability.	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)

Medium Distances (km)	variability *between* groups	0.0001466	1	0.0001466	7.4294e-07	0.99932
	variability *within* groups	3550.9023	18	197.2724		
	Total variability	3550.9025	19			

Table 4.2c: Analysis of Variance (ANOVA) of Medium Distances with Puissant method

	Source of the variability.	Sum of Squares (SS)	Degree of freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)
Medium Distances (km)	variability *between* groups	6.9627e-05	1	6.9627e-05	3.5388e-07	0.99953
	variability *within* groups	3541.5358	18	196.752		
	Total variability	3541.5358	19			

Table 4.3a: Analysis of Variance (ANOVA) of Long Distances with Bowring method

	Source of the variability.	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)
Long Distances (km)	variability *between* groups	0.97648	1	0.97648	0.0004289	0.98371
	variability *within* groups	40984.7718	18	2276.9318		
	Total variability	40985.7483	19			

Table 4.3b: Analysis of Variance (ANOVA) of Long Distances with Power series method

	Source of the variability.	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)

Long Distances (km)	variability *between* groups	7.9374e-07	1	7.9374e-07	3.4974e-10	0.99999
	variability *within* groups	40850.777	18	2269.4876		
	Total variability	40850.777	19			

Table 4.3c: Analysis of Variance (ANOVA) of Long Distances with Puissant method

	Source of the variability.	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F – Statistics (F)	p – value (Prob>F)
Long Distances (km)	variability *between* groups	0.00095111	1	0.00095111	4.2098e-07	0.99949
	variability *within* groups	40666.4972	18	2259.2498		
	Total variability	40666.4981	19			

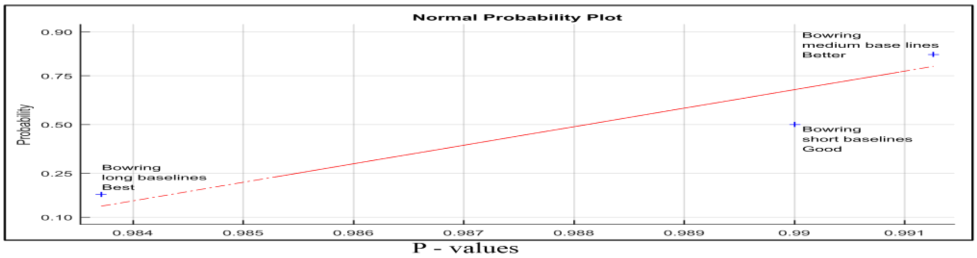

Figure 4.1: Graphic expression of the p – values of all base lines with Bowring methods

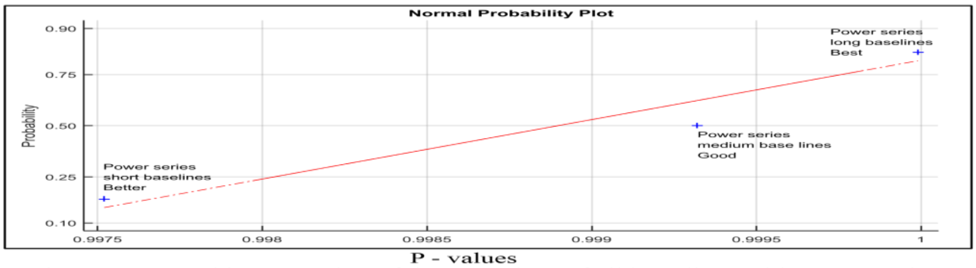

Figure 4.2: Graphic expression of the p – values of all base lines with Power series method

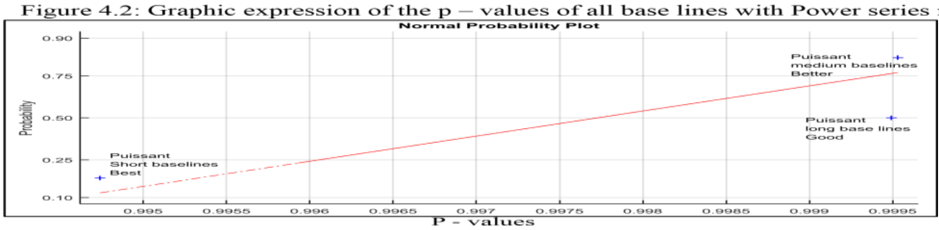

Figure 4.3: Graphic expression of the p – values of base lines with Puissant method

Table 4.4: Summary of comparative analysis of geodetic computation methods

Methods	**Short baselines**	**Medium baselines**	**Long baselines**
Bowring	Good	Better	Best
Power Series	Better	Good	Best
Puissant	Best	Better	Good

5.0 CONCLUSION

The research work has shown that geodetic information such as coordinates, distances and azimuths can be obtained using different types of formulas. The computations were carried out for both indirect and direct problems in all the methods. Bowring method is straight forward, easier and faster to computing because no iterations was required. Power series and Puissant methods were complicated due to iterations involved in computations of both indirect and direct problems. Iterations are needed in order to achieving very good results. The three methods of geodetic computation considered in this research work were actually good for computation of distances but each of the method was valid for a particular range of baselines. Bowring method is best used for long baselines computation. The accuracy of Bowring method becomes better as the baselines increases. Power series method is best used for short and long distances. Puissant method was valid for both short and medium baselines.

---- RECOMMENDATION

Some other geodetic computation methods such as Vincenty's method and Bessel's can be used to perform this type research work. The validity of the geodetic computation methods can also be researched into with respect to the increase in azimuths. Changes in azimuth can also have effect on the strength of each method putting into consideration the baselines range.
Apparently, further research is required to improve existing geodetic formulas or to develop new, less rigorous and better accuracy formula for geodetic computations.

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