Accurate PROJ parametrization of the Uniform National Projection System of Hungary (EOV)

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SUMMARY

We propose a new set of PROJ parameters for the definition of the Uniform National Projection System of Hungary (EOV) in GIS environments.
The EOV projection is an oblique Mercator map projection system, with two projection steps: first from the ellipsoid to a Gaussian sphere and secondly from the sphere to the cylinder. The projection in GIS applications is normally defined as a single step Hotine (or Swiss) Oblique Mercator projection with proper parametrization. However, this solution introduces a 1.5 mm average error due to the limitations of Swiss Oblique Mercator projection definition: the normal latitude of the ellipsoid (where linear distortion is minimal) and the latitude of the projection center (where the cylinder fits to the sphere) cannot be defined independently.
The concept of "transformation pipelines" started with PROJ 5.0.0 enabled us to define EOV projection in two steps (according to the official technical standard) eliminating this 1.5 mm error and enabling the PROJ definition to precisely fit to the standardized official solution.

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1. INTRODUCTION

In Hungary in the early 1970's a new geodetic datum the HD72 was introduced, using the IUGG1967 (aka: GRS67) ellipsoid. Simultaneously, a new map projection called Uniform National Projection System of Hungary (EOV) was defined for surveying and mapping applications. The EOV is an oblique conformal cylindrical projection also generally known as Oblique Mercator (Snyder, 1987). The meridian of the origin passes through the base point of Gellert Hill, Budapest. The ellipsoidal latitude of the origin was chosen to be close to the mean latitude of the country. The scale factor at the centre line is slightly below one to avoid the scale factor to exceed one above one hundred parts per million (ppm) in the main part of the country (Mugnier, 2017). All technical details including defining parameters and formulas are described in a technical standard released in 1975 (MEM OFTH, 1975).
EOV applies a double step projection process from ellipsoid through a conformal Gaussian sphere to the cylinder. This approach is almost identical to the one derived by Rosenmund for mapping of Switzerland (CH1903 also known as Swiss Oblique Mercator) (Rosenmund, 1903) but there is a slight difference. It has been shown by Molnár and Timár (2002) the EOV is like no other Oblique Mercator projection. Namely there is no functional relationship between the normal latitude of the ellipsoid (where the spherical projection is true to scale) and the latitude of the projection centre (where the cylinder fits to the sphere).

In standard cases of Oblique Mercator (CH1903 for instance) the latitude of the projection centre is derived from the normal latitude of the ellipsoid and the parameters of the reference ellipsoid. In the unique case of EOV the latitude of the projection centre was not calculated but it was arbitrary chosen without any functional link to the normal parallel. Presumably, it was defined this way because of historical heritage of a former national cylinder projection and to simplify the calculations with a chosen round value in the pre-computer times.

We can find many applications of Oblique Mercator as national map projections in many countries (Kennedy and Kopp, 2001; Chiriac and Vlascenco, 2015). It is significantly useful when conformality is desired and the mapping territories not having their greatest extent along neither meridians nor parallels, e.g. Alaska panhandle (Berry and Bormanis, 1970). It is also applied in the field of engineering surveying to connect small local geodetic systems to grid plane with a negligible distortion level in terms of scale factor (Takács and Siki, 2021).

As it is widely used, some form (e.g., Swiss, Laborde or Hotine) of the Oblique Mercator is included in many GIS systems. Very often in these systems (e.g., in QGIS, PostGIS, GRASS) the map projection related computations are completed using the open-source library called

PROJ. It is a generic tool for geodetic calculations. It provides cartographic projections, (even time dependent) geodetic transformations and coordinate conversions (PROJ, 2020).

There are three forms of Oblique Mercator in PROJ: Swiss, Hotine (Variant A and Variant B) and Laborde called *somerc*, *omerc* and *labrd* respectively. Unfortunately, none of them is able to precisely handle the unique case of EOV namely the differ of latitude of normal parallel from the latitude of projection centre as described above.

Currently in most of the GIS applications, based on the PROJ libraries, the EOV is approximated using the Swiss Oblique Projection formulas with unofficially derived parameters, and the EPSG:23700 code (IOGP, 2019) is identical with the work of Molnar and Timár (2002).

The average error of this approximation is about 1.5 mm in terms of the absolute numerical differences between the precise EOV and properly parametrized *somerc* formulas. Although the magnitude of the error is negligible on the level of a general GIS applications or even for more accurate geodetic tasks there is a way to eliminate this error in PROJ to obtain a coherent EOV solution in accordance with the national standard (MEM OFTH, 1975).

2. DATA

2.1. From ellipsoid to sphere

2.1.1 Closed form transformations

As the oblique conformal cylindrical projection is a double step projection, the first step is the conformal projection from the ellipsoid to an intermediate sphere. The equations for this transformation are below (MEM OFTH, 1975):

$$tg\left(45° + \frac{\varphi}{2}\right) = \frac{1}{k_1} \cdot tg^{k_2}\left(45° + \frac{\phi}{2}\right) \cdot \left[\frac{1 - e \cdot sin(\phi)}{1 + e \cdot sin(\Phi)}\right]^{\frac{k_2 \cdot e}{2}}$$

$$\lambda - \lambda_0 = k_2 \cdot (\Lambda - \Lambda_0)$$

where (φ,λ) are the spherical coordinates of the equivalent ellipsoidal (ϕ,Λ) coordinates. The coefficient *e* is the eccentricity of the GRS67 ellipsoid, and the coefficients $1/k_1$=1.0031100083 and k_2=1,0007197049 were chosen to minimize the distortion of this conformal projection along the ϕ_n=47°10’0.0000” predefined normal latitude.
The inverse of this projection can be calculated only with iteration.

2.1.2 Polynomial transformation

According to the computing capabilities of the 1970's, when the Uniform National Projection System was introduced, a polynomial series form for this transformation was also allowed by the technical standard (MEM OFTH, 1975),

$$\varphi = \varphi_n + \Delta\varphi$$
$$\lambda = \lambda_0 + \Delta\lambda$$

where φ_n=47.1222382777 is the spherical coordinate of the normal latitude and λ_0=19.0485717777 is the spherical longitude of projection origin. The spherical coordinate differences are calculated as:

$$\Delta\varphi = 0.99844601 \cdot \Delta\phi + 0.0024323 \cdot 10^{-5} \cdot (\Delta\phi)^2 - 0.000053 \cdot 10^{-10} \cdot (\Delta\phi)^3$$
$$\Delta\lambda = 1.0007197049 \cdot \Delta\Lambda$$

where $\Delta\phi$ and $\Delta\Lambda$ are in arcseconds and are calculated from:

$$\Delta\phi = \phi - \phi_n$$
$$\Delta\Lambda = \Lambda - \Lambda_0$$

The inverse of this transformation (from spherical to ellipsoidal coordinates) is also defined in the technical standard:

$$\Delta\phi = 1.00155641 \cdot \Delta\varphi - 0.0024436 \cdot 10^{-5} \cdot (\Delta\varphi)^2 + 0.000065 \cdot 10^{-10} \cdot (\Delta\varphi)^3$$
$$\Delta\Lambda = 0.9992808127 \cdot \Delta\lambda$$

using arcsecond values, for both latitude and longitude differences, and for both ellipsoidal and spherical coordinates.

2.2. From sphere to plane

In the second step, for the projection from the sphere the cylindrical plane, the so called auxiliary coordinates are calculated. These auxiliary coordinates are spherical coordinates on a rotated sphere, where the equator of the sphere is shifted to latitude of the projection centre: φ_0=47°06'0.00":

$$sin\varphi' = cos\varphi_0 \cdot sin\varphi - sin\varphi_0 \cdot cos\varphi \cdot cos(\lambda - \lambda_0)$$
$$sin\lambda' = \frac{cos\varphi \cdot sin(\lambda - \lambda_0)}{cos\varphi'}$$

These auxiliary spherical coordinates are used in the spherical form of Mercator projection:

$$x(= northings) = R \cdot m_0 \cdot \ln tg\left(45° + \frac{\varphi'}{2}\right)$$
$$y(= eastings) = R \cdot m_0 \cdot (\lambda' - \lambda_0)$$

where R=6379743,001m is the radius of the intermediate Gaussian sphere and the scale factor is m_0=0,99993.

Additional false easting and false northing values are added to these coordinates:

$$X^{EOV}(= northings) = 200000.0 + x$$
$$Y^{EOV}(= eastings) = 650000.0 + y$$

This addition proves, that easting coordinates are always greater than 400000m and northing coordinates are less than 350000m on the area of usage to avoid confusion.

3. METHODS

3.1. The pipeline operator

The 5.0.0 release of open-source PROJ library (PROJ, 2020) introduced the pipeline operator, that enhanced the flexibility of operations. Using the pipeline operator, a chain of operations, (e.g. transformation between geodetic datums, conversion between geocentric end ellipsoid coordinates, projection from ellipsoid to plane, inverse projections) can be defined and processed.
All calculations in this project were made on Windows 10 operating system, and using Proj 8.2.1. release.

3.2. Horner polynomial evaluation

The Horner polynomial evaluation, also introduced in 5.0.0 release of PROJ enables a polynomial style transformation between coordinates. The degree of polynomials can be set, and coefficients should be given by user. These coefficients are normally stored in a so called 'init' file. An example of this file can be seen on Fig. 1.

4. RESULTS

4.1. Horner polynomial evaluation

We defined a Horner polynomial type transformation for ellipsoid to spherical coordinate conversion.
The transformation was stored in a text file (later referred as 'proba', without file type extension). The unique identifier of the transformation was chosen: 'HD72GG'. We used a third order transformation, described in Section 2.1.
Polynomial coefficients are applied in a transformation using decimal degrees, so the coefficients of Section 2.1 were multiplied by 1, 3600 and 3600^2 respectively. For longitude transformation a simple first order transformation is defined, leaving higher order coefficient to zeros.
Inverse transformation coefficients were calculated from invers polynomial transformation (from spherical to ellipsoidal coordinates), and multiplying the coefficients by 1, 3600 and 3600^2 respectively.

```
<HD72GG>
+proj=horner
+ellps=GRS67
+fwd_origin=19.048571777777781,47.16666666666666
+inv_origin=19.048571777777781,47.122238277777775
+deg=3
+fwd_u=19.048571777777781,1.0007197049,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
+fwd_v=47.122238277777775,0.998446010000000,0.000087562800000,-0.000000068688000,0.0,0.0,0.0,0.0,0.0,0.0
+inv_v=47.16666666666666,1.001556410000000,-0.000087969600000,0.000000084369600,0.0,0.0,0.0,0.0,0.0,0.0
+inv_u=19.048571777777781,0.999280812702622,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
```

Figure 1. The content of a text file (an 'init' file in PROJ terminology) defining a Horner type polynomial transformation for ellipsoidal to spherical coordinates and for inverse direction.

4.2. The transformation using the pipeline operator

An example of the usage of our propose method can be seen on Figure 2., showing the content of an executable batch file.
The first line of the file executes the command line executable file 'cct' which is one of the PROJ executable functions, and offers a more flexible chain of operations.
The need for third and fifth lines of the batch, file containing +proj=eqc projections will be explained later in the discussion section.
The fourth line with '+init=proba:HD72GG' is referring the init file named 'proba' as discussed in the previous chapter, and this command is responsible for transforming ellipsoidal to spherical coordinates.
The last two lines of the batch file are responsible for applying the spherical form of the oblique Mercator projection to spherical coordinates. The +R parameter is the radius of the intermediate Gaussian sphere, and the +lat_0 is a spherical latitude of the projection centre. It is obvious, that no direct relation to the GRS ellipsiod can be found in this definition.

```
cct -vvv -z 0 -t 0 -d 10 ^
+proj=pipeline ^
+step +proj=eqc +a=57.295779513082323 ^
+step +init=proba:HD72GG ^
+step +proj=eqc +a=57.295779513082323 +inv ^
+step +proj=somerc +R=6379743.001 +lat_0=47.100000000
+lon_0=19.048571777777781 +k_0=0.99993 +x_0=650000.0 +y_0=200000.0
```

Figure 2. The content of an executable file that converts ellipsoidal coordinates of HD72 datum (GRS67 ellipsoid) to Unified National Projection (EOV) coordinates

5. DISCUSSION

In the previous section, the batch file example contains two strange lines, with '+proj=eqc +a=57.295779513082323' projection definitions. This projection definition is an equidistant cylindrical projection, that projects latitude and longitude to projected coordinates. The projected coordinates are calculated by converting respective ellipsoidal coordinates to radians and multiplying them by the radius of Earth. The +a parameter defines a radius that technically converts radians back to degrees. So, using this command, an ellipsoidal coordinate is converted to projected coordinate, that is numerically equals to the input longitude and latitude values. This operation was needed due to the limitation of the current PROJ release, i.e. the Horner polynomial evaluation algorithm accepts only projected coordinates. (The manual page of the Horner method however claims, that the algorithm accepts also geodetic coordinates as input. PROJ, 2020)

The polynomial type transformation of the ellipsoidal coordinates to spherical coordinates applied in this process has obvious drawbacks compared to the closed form transformation presented in Chapter 2.1. At extreme latitudes close to extent of the usage of EOV projection, the polyanomial and the closed form solution give significanly different results. This diffence is visualized on Figure 3. These differences are less than 1mm only up to 1,5 degrees north and south of the normal latitude (47°10'0.0") and dramatically increase toward the minimal (45.74) and maximal (48.58) latitudes (IOGP, 2019). This error can reduce the applicability of the method and requires another method (Horizontal grid shift or Triangulated Irregular Network based transformation) (PROJ, 2020) for the ellipsoidal to spherical transformation.

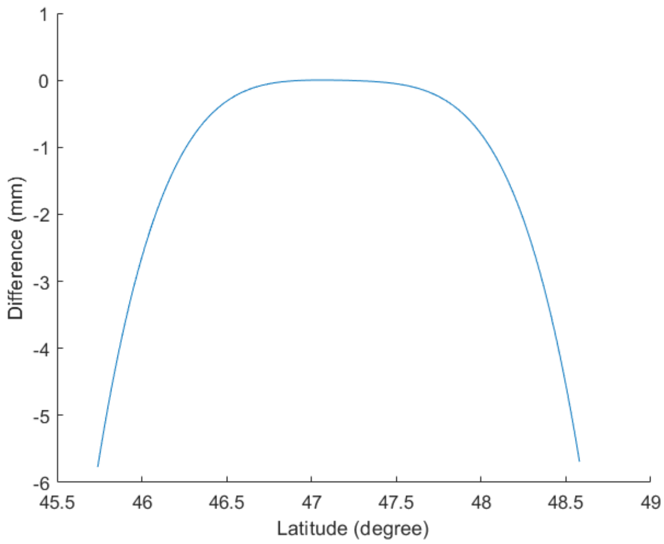

Figure 3. The difference between the closed form and polynomial type transformations used for ellipsoidal to spherical transformation. The differences are less than 1 mm only in the 1.5 degree range around normal latitude (47°10'0.00")

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BIOGRAPHICAL NOTES

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After graduating as geophysicist in 1994, he worked in the Space Research Group of Eötvös University, Budapest dealing with satellite imagery orthorectification and parametrization of map projections and geodetic datums for GIS software. Other research interests are related to laser scanning, Earth's mantle processes, muon tomography, and archaeometry.

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