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Statistical Evaluation of the B-Splines Approximation of 3D Point Clouds

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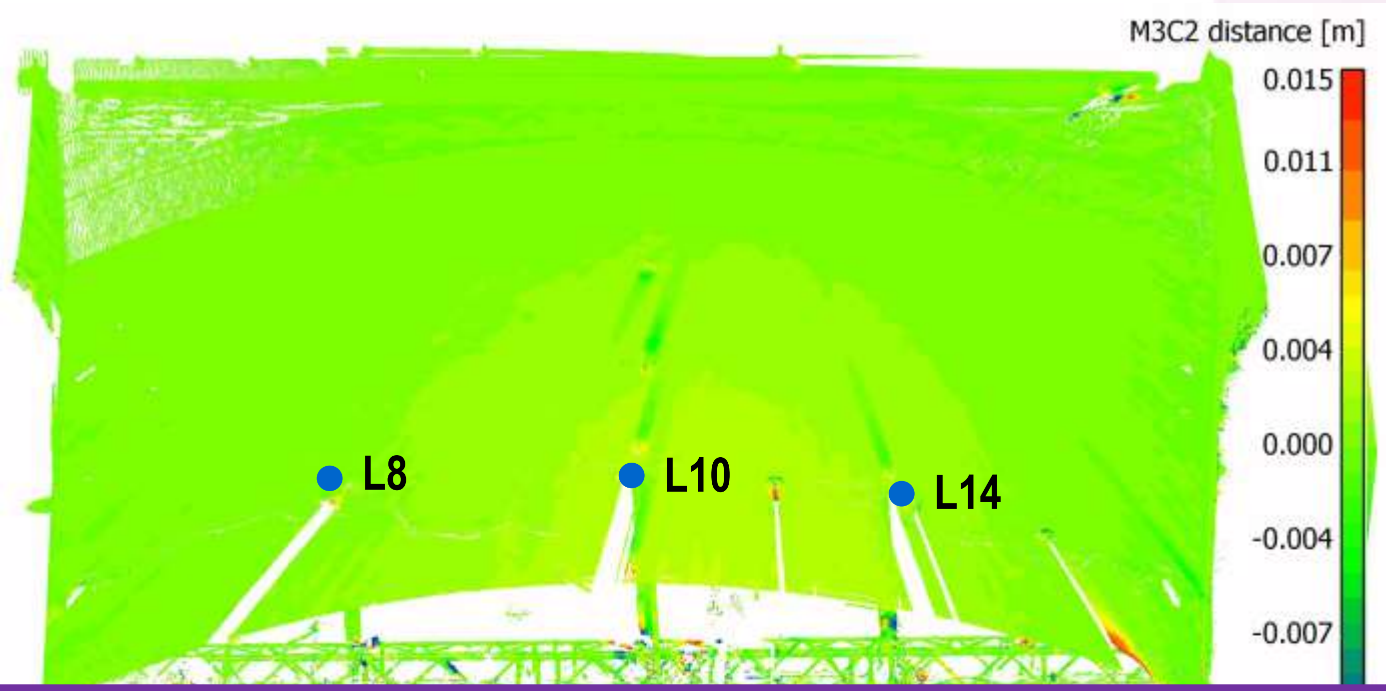
3D point cloud based monitoring of a masonry arch bridge

- Aim: Experimental investigations of the structural behaviour of the bridge by means of load testing

Data acquisition by means of laser scanner Z+F Imager 5006

- Epoch-wise 3D point cloud of the bottom side of the arch
- Data acquisition time per epoch approx. 7 minutes
- Evaluation in post-processing with Zoller+Fröhlich (Z+F) LaserControl, Scantra (technet GmbH), CloudCompare (www.danielgm.net/cc/)

3D point cloud to 3D point cloud differences (vertical comp.)



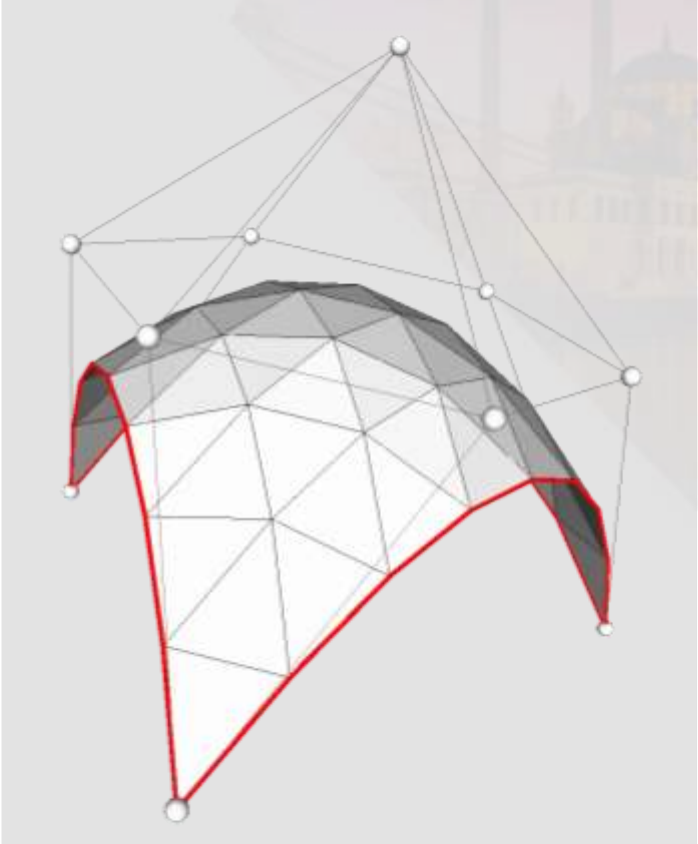
3D-pointclouds can be approximated by **free-form-curves and surfaces**, e.g. **B-splines**, in a robust way, so that deformations can be identified on the basis of the budget of uncertainty, even though data gaps and outliers can occur

Math. Basics - Parametric surface approximation: B-Spline

- Functional relation: piecewise polynomial function

$$\boldsymbol{S}(\bar{u},\bar{v}) = [x(\bar{u},\bar{v}), y(\bar{u},\bar{v}), z(\bar{u},\bar{v})]^T = \sum_{i=0}^{n}\sum_{j=0}^{l} N_{i,p}(\bar{u})N_{j,q}(\bar{v})\boldsymbol{x}_{i,j} \text{ with } \boldsymbol{x}_{i,j} = \left[x_{i,j}, y_{i,j}, z_{i,j}\right]$$

- Surface point: $\mathbf{S}(\bar{u},\bar{v})$
- Basis functions: $N_{i,p}(\bar{u}), N_{j,q}(\bar{v})$
- Control point: $\mathbf{x}_{i,j}$
- Location parameters: $\bar{u},\bar{v}$



Math. Basics - Steps to approximate a point cloud

1. **Model selection**: choose degree of basis functions p and q as well as number of control points n+1 and l+1
2. **Parametrization**: determination of location parameters $\bar{u}$ and $\bar{v}$
3. **Knot vector determination**: determination of knot vectors $\boldsymbol{U}$, $\boldsymbol{V}$
4. **Control point estimation**: estimation of control point net as parameters in a Gauss-Markov-model

Aim: Significance testing of different non-nested B-Spline approximations under variation of the number of control points and using different knot vector determination techniques

Testing non-nested regression models

Vuong test

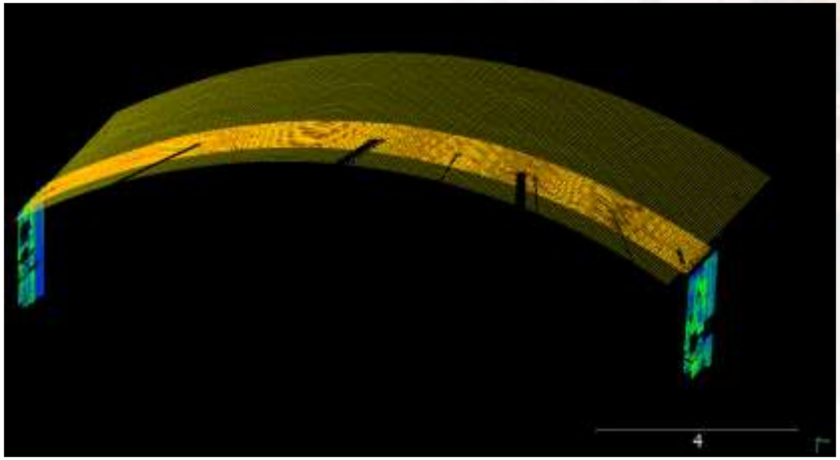
1. Run model I, saving the individual log-likelihoods
2. Run model II, saving the individual log-likelihoods
3. Schwarz Adjustment/Correction
4. Compute the test statistic based on differences (see paper)
5. Compute the $\alpha/2$ and 1- $\alpha/2$ quantiles of the N(0,1)

Clarke test

1. Run model I, saving the individual log-likelihoods
2. Run model II, saving the individual log-likelihoods
3. Schwarz Adjustment/Correction
4. Compute the test statistic based on differences and count the number of positive and negative values
5. The number of positive values is binomially distributed

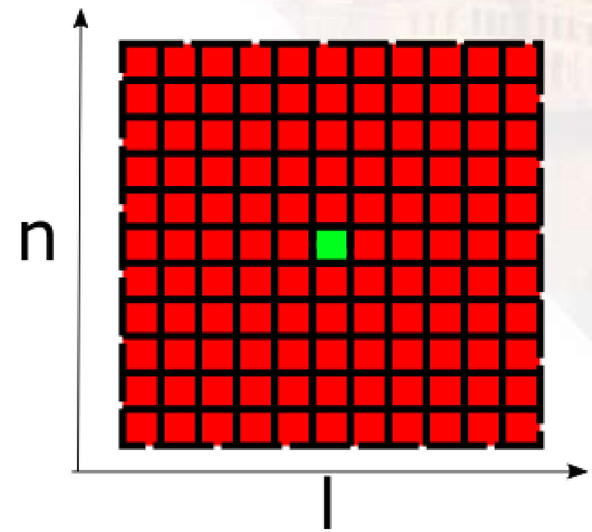
Case Study: Approximation of an arch bridge

- Section of the bridge arch without multiple changes of curvature
- Gridding the point clouds
  - x-expansion: 9.00 m
    - 400 cells → 2.3 cm
  - y-expansion: 14.00 m
    - 600 cells→ 2.3 cm
- Model Selection
  - Degree of the basis functions: p=3 and q=3
  - Number of control points
    - Variation of number of control points of 4 to 40 in both directions
    - Variation of knot vector determination technique



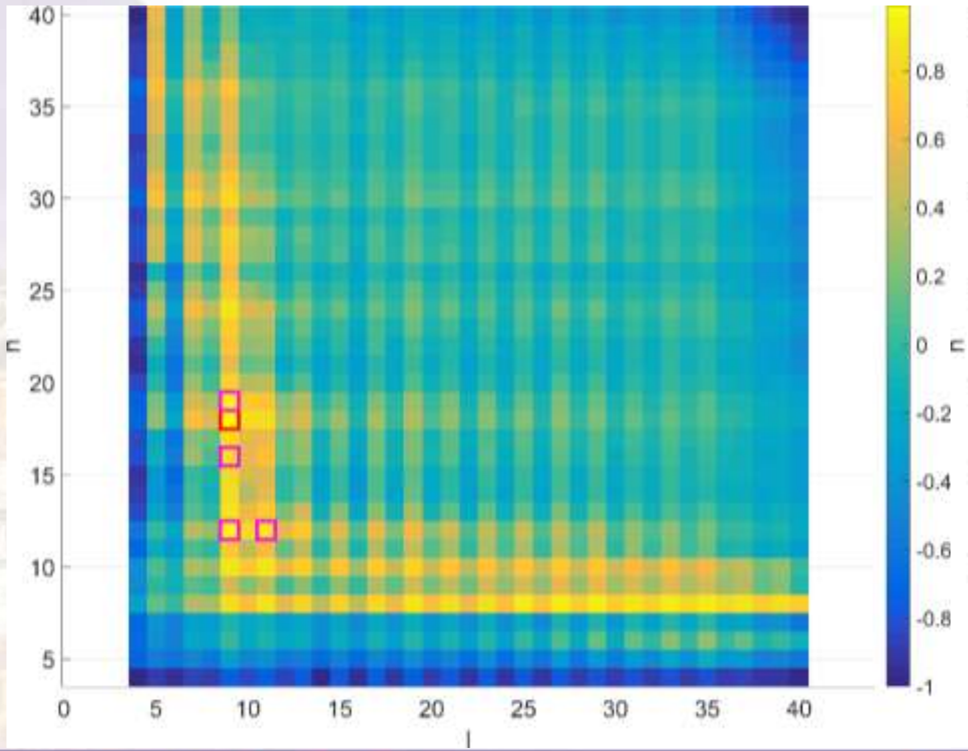
Test strategy

- Use Vuong's and Clarke's test to select the best B-spline model from 1369 possible combinations using two different knot vector determination methods
- Every model should be tested against all other models
  ➔ 2 million comparisons
- Developing a test strategy: competing models are only chosen from neighbourhood (5 rows and columns)
- Allocating score value for every model:
  - by +1 if this model is better than another model and by -1 if worse, otherwise 0
  - normalizing the score value
- Selecting the model with the highest scoring and test it against all other models

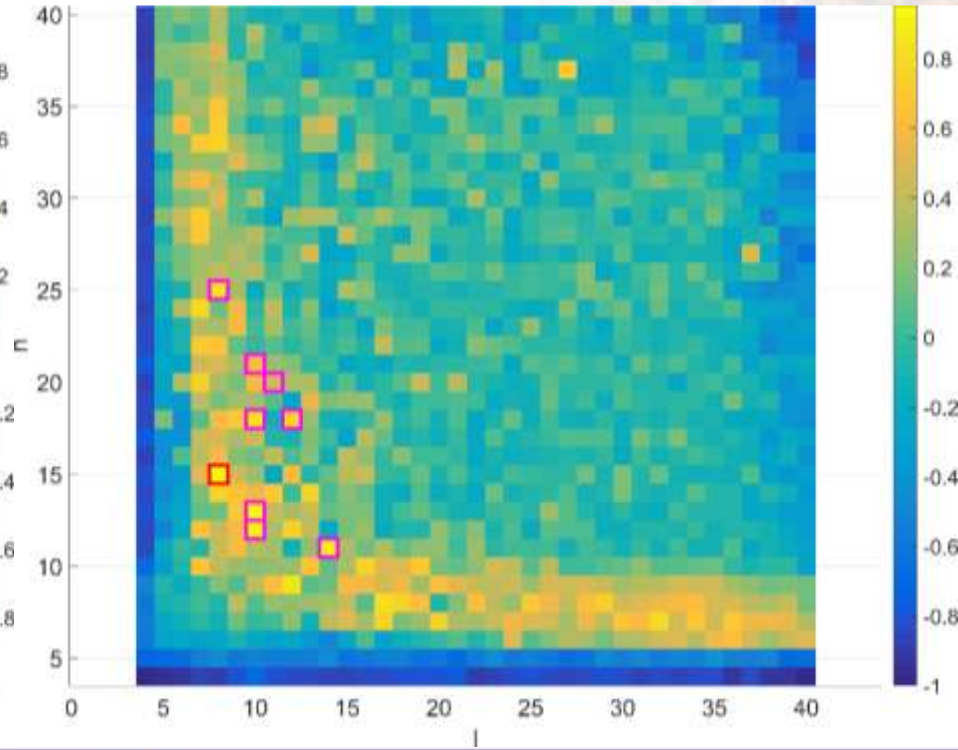


Results for Vuong's test for two different knot vector determination techniques

Knot placement technique

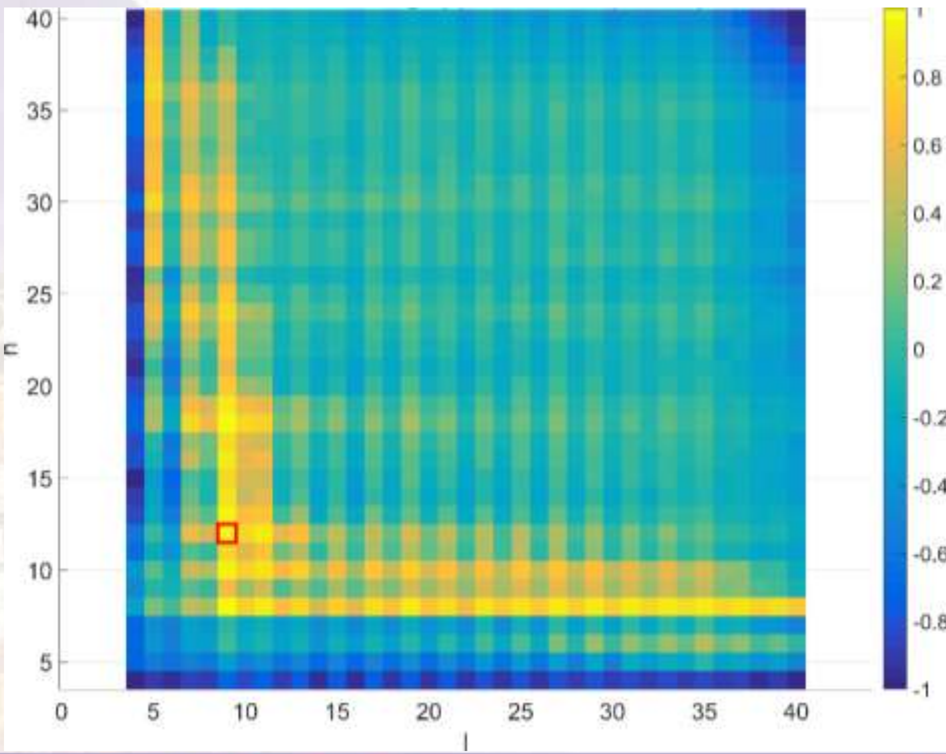


Monte Carlo technique

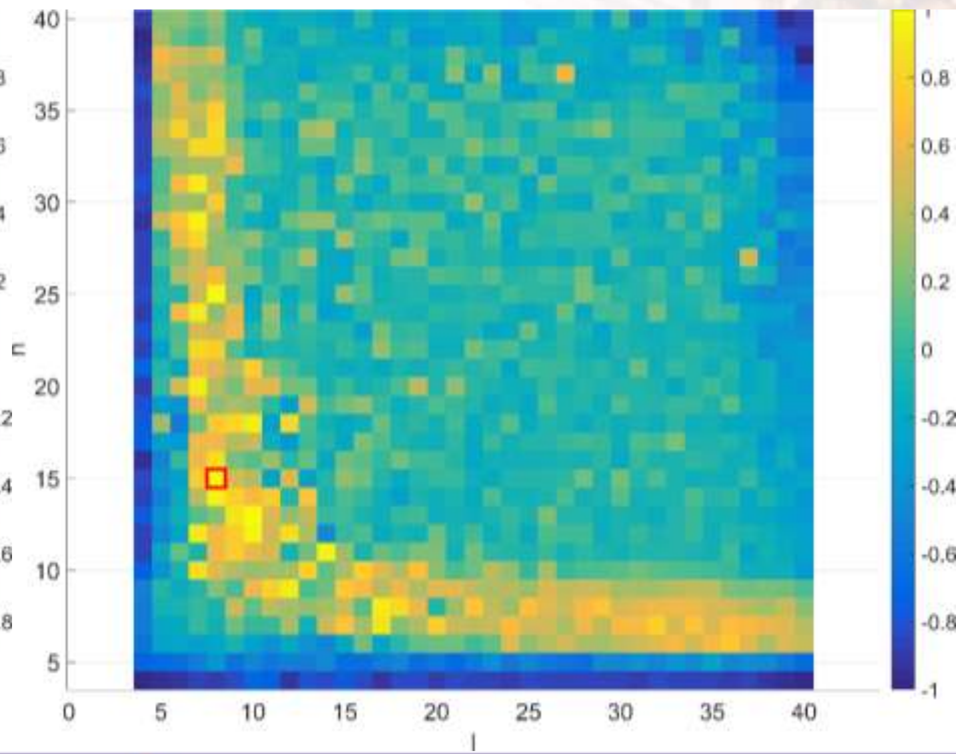


Results for Clarke's test for two different knot vector determination techniques

Knot placement technique



Monte Carlo technique



	Knot placement technique		Monte Carlo technique	
	n	l	n	l
AIC	39	27	37	27
BIC	12	9	15	8
Vuong	18 19 16 12 12	9 9 9 9 11	15 25 21 20 18 18 13 12 11	8 8 10 11 10 12 10 10 14
Clarke	12	9	15	8

Summary

- Using Vuong's and Clarke's tests for (non-nested) model selection in B-spline surface approximation
- These tests can detect **significant** model differences, in contrast to information criteria
- Both tests are based on likelihood-ratio and use Kullback-Leibler information criterion
- In many cases the Vuong test was not able to identify the best B-Spline model
- The Clarke's test approach, in contrast, can successfully identify one model over all other competing models