Knot Optimization for B-Spline Curve Approximation

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SUMMARY

Freeform curves with their possibility to approximate shapes from terrestrial laser scanner point clouds are investigated in this study.
We focus on B-spline curves which are able to capture the local behavior of the measured profile. Typically, the only parameter set, treated as unknowns by the approximation are the control points of the B-Spline. The second parameter set, the knots, which are part of the basis functions, are placed at stable locations. The approach with fixed knots leads to a linear system, but it intuitively restricts the B-Spline curve in its flexibility. Estimating the control points and the locations of the knots at the same time succeeds in full flexibility of B-Splines and optimizes the approximation. The accrued system of equations is highly non-linear. To enhance the convergent behavior, constraints and adequate starting values are necessary. The arbitrary values of the knots are chosen with a new bottom up method, starting with the minimum number of knots and adding one knot in each iteration step at a particular curve sections (span). The decision to insert a knot at a specific location, is based on the analysis of the residuals in each span.
The improvements are shown by comparing the results obtained in the linear approach with fixed knots and the non-linear case where control points and the knots are treated as unknowns.

KURZFASSUNG

Freiformkurven können zur Approximation von Punktwolken aus terrestrischen Laserscans genutzt werden.
In dieser Untersuchung werden B-Spline Kurven eingesetzt, die je nach Parameterwahl lokale Gegebenheiten in einer globalen Approximation erfassen können. Typischerweise werden bei einer Approximation von B-Splines die Kontrollpunkte in einem linearen Modell geschätzt. Ein weiterer Parametersatz sind die Knoten, mithilfe derer die Basisfunktionen erstellt werden. Die gemeinsame Schätzung der Knoten mit den Kontrollpunkten ergibt ein hochgradig nichtlineares Gleichungssystem. Die volle Flexibilität zur lokalen Anpassung wird erst durch die Schätzung beider Parametergruppen erreicht. Zur Stützung des nichtlinearen Gleichungssystems werden Bedingungsgleichungen eingeführt und Näherungswerte für die Knoten mit einer neuen Methode ermittelt. Diese basiert auf den Residuen der linearen Schätzung der Kontrollpunkte, die in Teilbereichen der Kurve, den sogenannten Spans, analysiert werden. Begonnen wird die Approximation mit der Minimalkonfiguration an Knoten bis zu einer zuvor definierten Maximalanzahl.
Die im neuen Ansatz erzielte Verbesserung wird durch den Vergleich der Ergebnisse aus der Schätzung der Knoten und der Kontrollpunkte demonstriert.

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1. INTRODUCTION

Surface-based metrology, like terrestrial laser scanner (TLS), needs new surface-based evaluation methods. These approximation methods are one of the main challenges making the information of 3D point clouds suitable and taking benefits from the redundancy. Freeform curves and surfaces are promising approximation methods to create parameterized curves and surfaces for further evaluation steps, like shape information for structural analysis of built objects (Schmitt et al., 2013). In the past, research has shown that freeform shapes significantly improve the approximation quality, compared to approximations with geometric primitives, e.g. (Schmitt et al., 2014).
In this paper, B-Splines are used to approximate TLS profiles. In the past, this was only done by estimating the control points of the B-Spline. Another essential parameter set for B-splines are the knots. The optimization of the knot locations leads to a nonlinear system of equations. Improved arbitrary values are needed to solve the system. The accomplishment of this issue is a challenge and is done by a new method presented in this paper. The optimization scheme is performed subsequently. Afterward the method is evaluated against the state of the art techniques on simulated data. Furthermore, the algorithms are tested on a real TLS 2D-profile data set. Extended parts of this research were already published in (Schmitt and Neuner, 2015).

2. SPATIAL APPROXIMATION WITH B-SPLINES

For the spatial approximation of the TLS 2D profile data, B-Spline curves are used. They were designed from De Boor (DeBoor, 1978) and de Casteljau described in (Piegl and Tiller, 1997) and applied especially in CAD designs and construction of cars. The challenge in this paper is to use them in the opposite way for approximating existing curves and surfaces, based on single points. The advantage of the B-Splines is their flexibility in matching most of the curves with respect to their local behavior. The local behavior of the curve is controlled by the distance between the knots, the span length. The smaller the spans the more curvature changes / local details can be modeled. The variable p defines on the one hand the degree of the single basis function and on the other hand the number of linear combined basis functions. Further parameters of the B-Splines are the control points, CP_i, which can be stated as weights for each basis function $N_{i,p}$ and obs, the homologues parameters at the curves of the observations, OBS (coordinates in *X*, *Y* direction)

$$C\ (obs) \ = \ \sum_{i=0}^{n} N_{i,p}(obs)\ CP_i \ , i \ = 0 \dots n \tag{1}$$

where n + 1 is the number of control points.

3. STATE OF THE ART

B-Spline curve approximation from 2D TLS-profiles is a new field in engineering geodesy that refers to the deformation analyses, as shown in (Schmitt et al., 2013), (Neuner et al., 2013).The characteristic of a TLS point cloud is normally its high point density and its homogeneous distributed points without gaps.
The standard method for the parameterization of the OBS is the uniform distribution, where the number of points is distributed with equal space along the curve, which is mostly not the case for measured data. A second method based on the chord length between the OBS. It roughly approximates the arc length on the curve. A third one is the centripetal method, which contains the centripetal acceleration and curvature. Lee describes in (Lee, 1989) all three methods in detail, with an evaluation of them to each other and gives recommendations on deploying them, see also (Piegl and Tiller, 1997).
Estimating the CPs is done by solving the linear system of equations (1).
Separating the influence between the degree and other parameters of the estimation results is often hard. That’s why the arbitrary value for the degree of the basis function is chosen empirically and set to a fixed value with respect to the experience about the observations, the further applications and the requirements to the curve continuity.
The last parameters are the knots, which are necessary for the formulation of the basis functions. There are two estimation issues referring to the them: first the number of knots and second their location inside the knot vector, the most complex one. For the number of knots, the AIC element provides promising results, shown in (Lindstrom, 1999) and (Harmening and Neuner, 2014). Regarding the location of the knots the uniform distribution performs poorly on heavy irregular curved data sets. Other algorithms depending on the distribution of the observations, like the basic method described in (Piegl and Tiller, 1997) and its extended version in (Piegl and Tiller, 2000) by considering the Schoenberg-Whitney condition (Schoenberg and Whitney, 1953). A further algorithm is the section midpoint strategy, which sets a new knot in the middle of a span, or at the position with the highest residual to an arbitrary curve. Others techniques uses the arbitrary length and curvature for the knot placement, e.g. (Razdan, 1999), (Park and Lee, 2007). Estimating the location of the knots during the approximation leads to a highly nonlinear optimization problem as mentioned before, which is denoted as the “lethargy” and extensively described in (Jupp, 1975).

4. IMPLEMENTATION

The developed approximation performed here, describes an iterative two-step estimation of the control points and knot locations. It is focused on the optimization of the geometric parameters itself. This is the reason why the identity matrix was used in the stochastic model. The following schema shows the individual steps of the method:

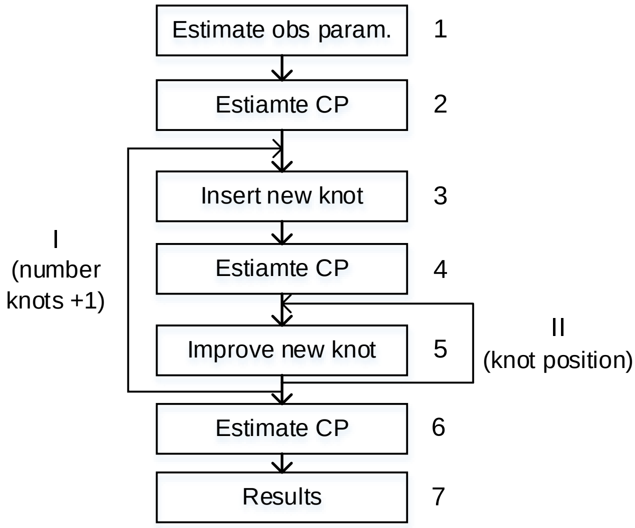

Figure 1: Algorithm sequences

In the first step the parameters (obs) for the measured 2D points are estimated with the methods uniform or centripetal appropriated to the datasets. In the 2nd step the CP for the minimal configuration of the B-splines, denoted as Bézier curve, with p+1 basis functions, are estimated using the linear least squares Gauß-Markov model (1).

For the B-Splines used here, the first and last control point fit the first and last observation respectively. These restrictions and the composition of the minimal knot vector with p+1 zero values and p+1 one values, imply that the slope of the line between the first and second CP and between the last and penultimate CP is equal to the slope of the curve at the start and end point. The restrictions are necessary to prevent oscillations of the curve at these points and applied to all approximations, which are mentioned here.

Based on the residuals obtained from the CP estimation the new knots and their locations are estimated iteratively within the loop I (step 3 – 5).

In each span two cumulative sums of squared residuals are calculated. One starting from left to right and second starting from right to left. The squared sum can be understood as the potential energy (pt) of the residuals inside a span. The position (ppt) inside a span, at which the subtraction of both versions, pt left-right and pt right-left, is zero, is the position, which is needed to reduce the pt with the highest effect. The value of the cumulative sum at this point is denoted as (ept). The new knot will be inserted in the span with the highest ept value to reduce the highest pt in the curve. At each iteration step of loop I, only one new knot is inserted. In step 4 the position of the new CP, which occurs due to the new knot insertion, is estimated with the model (1). Afterwards only the location of the new knot is improved by a nonlinear iterative estimation (loop II) using a restricted linearized Gauss-Markov model (step 5). In this model only the position of the new knot is unrestricted while all other knots are fixed by constraints.

In step 6, the new locations of the CP's are estimated using the optimized new knot position. The quality parameters of the results are obtained in step 7, after the convergence of the nonlinear model (loop II) and the insertion of the maximum number of knots (loop I). A simultaneous global optimization of the curve parameters, the CPs and the knot location cannot be realized yet due to reasons shown in the next chapter.

5. SIMULATED DATA

Simulation studies performed in this chapter aim to validate the developed method and to infer its behavior. Therefore, the test dataset was generated referring to the TLS profile analyzed in chapter 6. The noise is processed from the normal distribution. The number of sample points, 5000, are chosen as high as the $\sigma_{apost.,lin.,}$ obtained from the linear approximation in step 2, is equal to the value $\sigma_{apost.,sim}$, used by preparing the simulation data, e.g. Table 1. The sample points are distributed uniformly with a point to point distance of 0.015.

Table 1: B-Spline parameters for simulated data

Type	Values
Knot vector	{0, 0, 0, 0.10, 0.15, 0.30, 0.55, 0.60, 0.75, 0.90, 1, 1, 1}
Number of knots	13
Number of basis functions / CP	10
Degree p	2
Dimension	2 (X,Y)
Noise – Variance	0.005^2
Number of sample points	5000

Different estimation scenarios were applied on the simulated curve; these are summarized in Table 2. The degree of the basis function and the number knots were set equal to the ones of the designed B-Spline. All three above-mentioned methods of parameterization were applied to the observations. A comparison of the obtained results is given in Table 3. For further processing the parameters were not recalculated. Instead, they were taken directly from the simulated B-Spline

5.1 Results

The convergent criteria, which was reached after the second iteration, shows that the algorithm and the digits of the values are precise enough for our application.

Table 2: Approximation results compared to each other – on simulated data (T_F = Test value, $F_{9984,\,9984,\,0.05}$ = Quantil of the fisher distribution)

<table>
<tr><th>Noise</th><th>Method</th><th>$\sigma_{apost.}$</th><th>T_F</th><th>T_F</th><th>$F_{9984,\,9984,\,0.05}$</th></tr>
<tr><td>Yes</td><td>New method nonlinear</td><td>0.11</td><td></td><td rowspan="2">1.09</td><td rowspan="3">1.03</td></tr>
<tr><td>Yes</td><td>New method linear</td><td>0.12</td><td rowspan="2">1.53</td></tr>
<tr><td>Yes</td><td>Basic</td><td>0.18</td><td></td></tr>
</table>

The results of three methods are compared:

- The developed method - nonlinear (including loop II),
- The developed method - linear (without loop II),
- The basic/state of the art method.

The arbitrary knot placement strategy shows “significantly” better results than the traditional one, in case of uncorrelated variances. The standard deviation is taken from the linear least squares with the CP’s as unknowns.

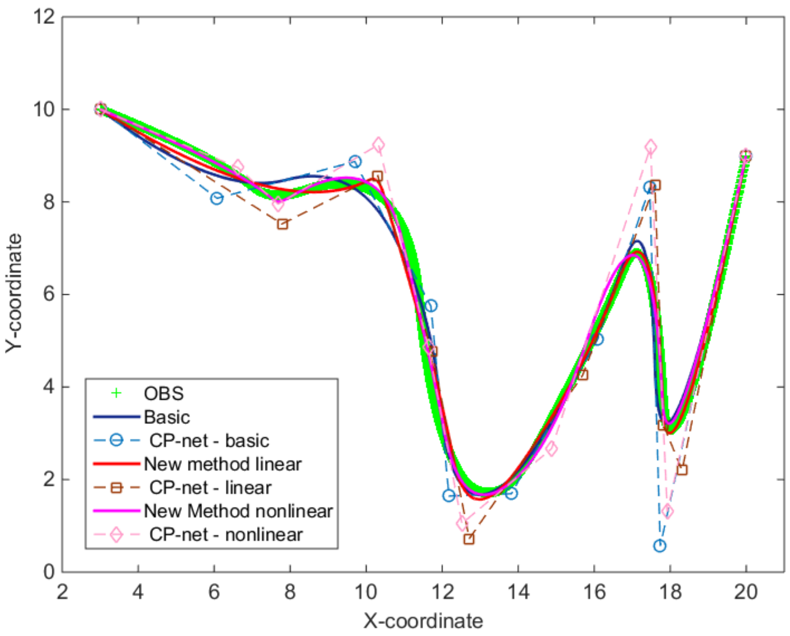

Figure 2: Result of the B-Spline curve approximations with the simulated data

Figure 2 shows the curves obtained with the methods from Table 2. The highest residuals are at the beginning where the developed method produces in the linear case sharp peaks. The nonlinear method is closer to the OBS, but with a high curvature turn. The traditional method leads to a complete shift at the beginning, which can be interpreted with a shift of the knots at that part. No systematic effects can be observed from the distribution of the CPs. Hypothesis tests cannot be performed correctly on the results, because in our case the functional model of each method changes. The reason is the different knot vector, which

affect the calculated basis functions and the affiliation of the obs to the basis functions. This aspect justifies the sequential knot optimization: the updates of the knot position in loop II are related to the actual functional model and not to the new one that accounts for changes of the basis-function allocation and the relation of the obs to the basis functions.
The reason for using the parameters of the simulated B-Spline itself is given by the differences between the three mentioned methods.
Figure 3 shows the differences between the three parametrization methods:

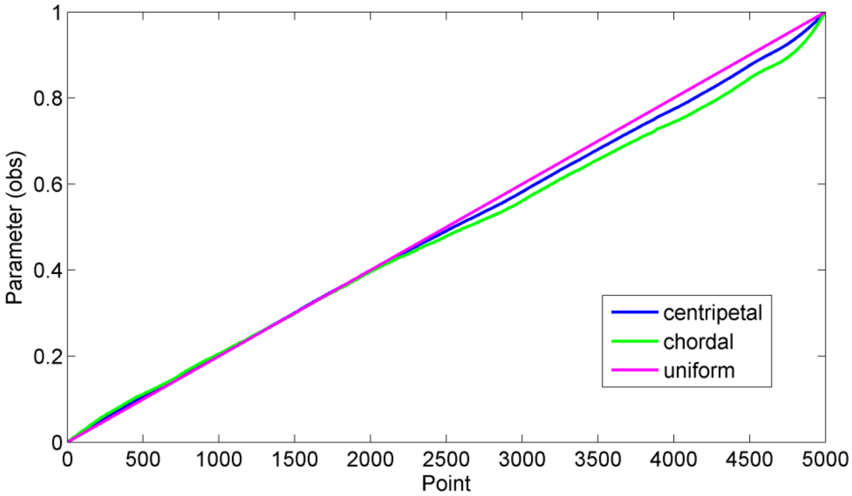

Figure 3: Parameters of the obs, calculated with different methods

The parameters differ significantly in the part of the curve with high curvature. As a result of this all known parameterization methods allocate insufficient obs between the spans for the further approximation algorithms. This can be noticed in Table 3, where all linear approximations of the control points significantly differ from the ones obtained for the uniform/simulated case.

Table 3: Results of linear least squares with different obs values (T_F = Test value, $F_{9984, 9984, 0.05}$ = Quantil of the fisher distribution)

<table>
<tr><th>Noise</th><th>Method</th><th>$\sigma_{apost.}$</th><th>T_F</th><th>T_F</th><th>T_F</th><th>$F_{9984, 9984, 0.05}$</th></tr>
<tr><td>Yes</td><td>Simulated (uniform)</td><td>0.0052</td><td></td><td rowspan="2">56</td><td>40</td><td rowspan="3">1.034</td></tr>
<tr><td>Yes</td><td>chordal</td><td>0.2932</td><td rowspan="2">1.38</td><td></td></tr>
<tr><td>Yes</td><td>centripetal</td><td>0.2120</td><td></td><td>40</td></tr>
</table>

6. MEASURED DATA

The measured data are produced from a TLS profile scan, with a total length of 14 m. The Z coordinate needs to be sensitive. Therefore it is trend reduced and scaled to [mm]. The Y coordinate is in [m] and represents the step size of points. The point density was reduced from

12293 to 5000 points in order to emphasize the impact of the step size. The parameters for the B-Splines are given in Table 4.

Table 4: Parameters for the B-Spline approximation of the measured data

Type	Values
Number of knots	22
Number of basis functions / CP	19
Degree	2
Dimension	2 (Z,Y)
Sample points	5000 / 12293

The curve parameters allocated to the OBS are calculated with the centripetal method. It has generated the best result compared with the other parameterization methods.

6.1 Results

The estimation results are summarized in Table 5. As can be seen the new arbitrary knot estimation method is better than the estimation of the CPs only in the linear model. However, the decrease of the standard deviation of the CP estimation is not as strong as in the case of the simulated data set. The variation of the number of knots, degree of the basis function and the number of OBS results in the order from the best result by the new method - nonlinear to the basic method, in Table 5. The “significant” changes need to be carefully interpreted with the difficulties of the Hypotheses test in mind, mention in the result of the simulated data.

Table 5: Approximation results compared to each other – on measured data (T_F = Test value, $F_{f1, f2, 1\text{-alpha}}$ = Quantil of the fisher distribution)

<table>
<tr><th>Num. Points</th><th>Method</th><th>$\sigma_{apost.}$ [mm]</th><th>T_F</th><th>T_F</th><th>$F_{9940, 9940, 0.05}$ / $F_{24526, 24526, 0.05}$</th></tr>
<tr><td rowspan="3">5000</td><td>New method non linear</td><td>0.68</td><td></td><td rowspan="2">1.07</td><td rowspan="3">1.03</td></tr>
<tr><td>New method linear</td><td>0.73</td><td rowspan="2">1.33</td></tr>
<tr><td>Basic</td><td>0.97</td><td></td></tr>
<tr><td rowspan="3">12293</td><td>New method non linear</td><td>0.70</td><td></td><td rowspan="2">1.07</td><td rowspan="3">1.02</td></tr>
<tr><td>New method linear</td><td>0.75</td><td rowspan="2">1.25</td></tr>
<tr><td>Basic</td><td>0.94</td><td></td></tr>
</table>

Figure 4 shows the different B-Spline curves after the knot estimation with the three methods in the case of 5000 sample points.

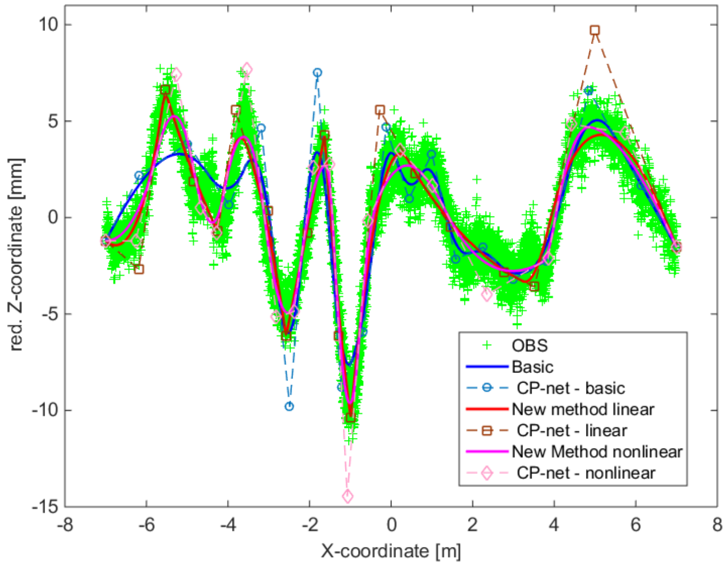

Figure 4: Result of the B-Spline curves approximation on measured data

Especially at the beginning of the curve, the developed methods lead to closer fits to the OBS. In the section between 0 m and 4 m, the approximation with the basic method is more detailed. These oscillations appear also in the case of estimation with the new methods by a higher number of knots. The differences between the linear and the nonlinear case of the new method, is the smoothness of the curve. While in the linear case the curve is approximated with sharp peaks and the nonlinear model leads to rounded peaks and a smoother curve.

7. CONCLUSION

The developed method for the knot estimation improves the approximation of TLS profiles with B-Splines. The algorithms were validated with simulated data and applied on real data. They show “significantly” better results than the basic method.
When estimating all knot locations at once the algorithm becomes unstable. Despite the functional problems during the nonlinear iteration, the algorithm converges when estimating only one knot by using small update values. Estimating only one knot location at each iteration step of loop II leads to good results also in case of higher knot numbers. Oscillations like in the case of polynomials with higher degree didn’t occur when the estimation was performed up to 23 knots on the simulated dataset in order to reach the standard deviation original variance.
The influence of the parameterization tends to be higher than the influence of the correct number of the knots. Whereby, similar approximation results were achieved by different numbers of OBS.
Extending the sequential nonlinear model to a global estimation of knots and CP’s is aimed in future research.

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BIOGRAPHICAL NOTES

Claudius Schmitt, M.Sc. received his Master in Geodesy and Geoinformation at the University of Applied Science Mainz in 2011. He has been started his doctoral program at the Geodetic Institute at the Leibniz Universität Hannover 2011-2014 continouing it at the Department for Geodesy and Geoinformation – Engineering Geodesy at the TU Wien. His main research interests are: Modeling of laser scan point clouds for structural analysis.

Hans Neuner, Prof. Dr. studied Geodesy at the Faculty of Geodesy, Technical University of Civil Enginnering Bucharest, Romania. In 2007 he received his Dr.-Ing. from the Leibniz University in Hanover, Germany. Since 2013 he is a professor for Engineering Geodesy at the Technische Universität Wien. The main research areas are the development and analysis of areal measurement and processing techniques, deformation monitoring and time series analysis. He is active in international and national professional unions and scientific associations and a member of the editorial board of the Journal of Applied Geodesy.

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