Some Aspects on Basic Gravimetric Network Adjustment

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SUMMARY

In this paper are summarized and analysed some types for estimation of precise relative gravimetric networks measured simultaneously with two or more gravimeters. The studies are based on functional model with preliminary calculation and elimination of the drift. Different types of stochastic models of measurements are examined. Some robust methods for estimation of measurements are presented.

The presented theoretical approaches are applied to the estimation of the Basic Gravimetric Network of Republic of Macedonia. The comparison between the results from the least squares adjustment of stochastic models is based on models of equal weights, weights proportional to the standard deviations of the measurements, weights reciprocal to the time differences, and on a model accounting the relation between the measurements in each gravimetric loop. Robust estimation methods are examined and the Danish method is chosen as the most appropriate for relative gravimetric networks. Calculations and estimation methods are applied for each gravimeter separately and combined models are made.

SUMMARY

В статията са разгледани и анализирани подходи за оценка на прецизни релативни гравиметрични мрежи, измервани с два и повече статични гравиметри. Изследванията са основани на функционален модел с предварително определяне и елиминиране на дрейфа на нулата от измерените гравиметрични разлики. Разгледани са различни стохастични модели на измерванията. Представени са някои от робастните методи за оценка на измерванията.

Предлаганите теоретични подходи са приложени за оценка на първокласната гравиметрична мрежа на Република Македония. Извършено е сравнение между получените резултати от изравнение по МНМК при стохастини модели с еднакви тежести, тежести пропорционални на стандартното отклонение на измерванията, обратно пропорционални на времето, както и на модел, отразяващ зависимостта между измерванията в гравиметричния рейс. Изследван е подход с прилагането на робастен метод, като за най-подходящ за прилагане е избран Датският метод за оценка на измерванията. Изчисленията и методите за оценка са приложени за измерванията със всеки гравиметър поотделно, след което са съставени комбинирани модели.

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1. INTRODUCTION

Precise gravimetric network adjustment in realization of First class national or regional gravimetric networks is discussed in this paper. These networks are realizing and regularly distributing gravimetric reference system over given territory.
Processing of measurements in contemporary precise gravimetric networks is based upon given regulations in realization. These preliminary requirements are set in the procurement for establishment of the precise gravimetric network. Main characteristics forming mathematical model of the relative gravimetric network measurements are:

- Configuration of network;
- Scheme of gravimetric measurements;
- Type of used relative gravimeter(s) – Technical specifications
- Number of used gravimeters.

Realization of adjustment (estimation of functional and stochastic parameters) is applied in four main stages:

- Defining of mathematical model of measurements;
- Preliminary and post-processing estimation of measurements;
- Assessment of optimal method for estimation;
- Assessment of quality of mathematical model and optimization criteria.

The goal of presented paper is to highlight some specific moments in process of estimation, depending of main circumstances on three main stages. Practical results and analyses are given from estimation of First order gravimetric network of Republic of Macedonia.

2. MATHEMATICAL MODEL OF RELATIVE GRAVIMETRIC MEASUREMENTS

The two components of the mathematical model are functional and stochastic model of measurements in network as is well known. Mathematical model for relative gravimetric measurements in precise gravimetric networks is not uniquely defined. It could be formed depending on listed main characteristics of network and chosen preliminary processing.

2.1 Functional model

2.1.1 <u>Parameters</u>

The functional model describes relations between the measurements in network and unknowns. For relative gravimetric measurements possible parameters in observation equations are three main groups:

(1) parameters for gravity accelerations;
(2) parameters for drift model;
(3) parameters for calibration model.

The first one is always included in the model. The second and third groups - the drift and calibration parameters - could be preliminary eliminated.

2.1.2 Observation equations

Commonly used functional model is based on **measured gravity differences** ΔG_{ij}. The general form of the adjusted value with measured gravity difference is:

$$\Delta G_{ij} = G_j - G_i + D(t_0, t_i, t_j) + K(O_i, O_j) + \delta S_j - \delta S_i . \qquad (1)$$

The other possible functional model is that based on **gravity readings of measurements** (real value of observation in milligals):

$$G_i = O_i + D(t_0, t_i) + K(f_o, O_i) + \delta S_i , \qquad (2)$$

where

G_i, G_j – *adjusted values for gravity accelerations in gravity stations i and j*;

t_0, t_i, t_j – *time moments in gravity loop*;

O_i, O_j – *gravity readings, corrected for systematic effects*;

f_0 – *initial factor for converting in milligals*;

$\delta S_i, \delta S_j$ – *systematic parameters for gravity measurements in gravity stations i and j*

The parameters in functional model describing relative gravimetric measurements could be presented in three main groups:

- Parameters for gravity acceleration values - G ;
- Parameters of drift function - $D(t)$, where t is time moment;
- Parameters of calibration function - $K(O)$, where O is gravity reading.

The systematic parameters are evaluated as insignificant from Sevilla et.al. (1990).

Frequently used corrections for systematic effects are *earth tide correction, polar motion correction, atmospheric correction and elevation correction*. In case of additional data could be introduced and other corrections like hydrological effect, human activities and etc. They are introduced preliminary and are not included in the functional model of measurements.

The adjusted values for gravity accelerations in formula (1) could be present with their approximate values (G_i^0, G_j^0) and unknown parameters $(\delta G_i, \delta G_j)$:

$$\Delta G_{ij} = \delta G_j - \delta G_i + D(t_0, t_i, t_j) + K(O_i, O_j) + G_j^0 - G_i^0 + \delta S_j - \delta S_i \quad (3)$$

This is the general form, which could be reduced with preliminary introduction of drift and/or calibration parameters.

Most frequently used are observation equations in form (1) with gravity differences. In that case there are eliminated some parameters (Sevilla et.al., 1990). One of these parameters is accumulated drift parameters in zero moment $(D(t_0))$ when drift model is linear. Other parameter that could be eliminated is parameter for unknown possible additional systematic effects (δS). The next eliminated parameter is the initial calibration value (f_0). The main benefit is that with use of gravity differences is reduced the number of calculations and drift model could be presented with low degrees (Torge, 1989). The same advantages are concerned for calibration function.
But application of equations in form (2), based on gravity readings, is suitable in complicated and various schemes for gravity loops in network.

2.2 Stochastic model

The stochastic model is represented by variances of measurements or their covariance matrix. The structure and contents of stochastic model strongly depends on formed functional model. Usually stochastic model describes the accuracy of measurements and their algebraic correlation. The algebraic correlation depends on configuration of network and scheme of measurement. Between relative gravimetric measurements besides algebraic correlation there exists and physical correlation too. The main reasons for physical correlation are incomplete reducing of drift and residual influence of systematic errors (Torge, 1989). The modelling of physical correlation could be made in two different ways. The first is based on research of measurements residuals or errors to determine empirical correlation coefficients (Becker, 1984). The second is based on forming of measurement weights as function of time. This approach is used in IGSN-71 (McConneil and Gantar, 1974).

Types of stochastic models commonly used in practice are:

a. ***With diagonal or non-diagonal structure***;

b. ***Depending on weights of measurements***;

(1) **Equal weights**:

$$p_{\Delta g} = 1 \tag{4}$$

(2) **Reciprocal to time of measurement** (Δt):

$$p_{\Delta g} = \frac{c}{\Delta t},\ p_{\Delta g} = \frac{c}{\Delta t^2} \tag{5}$$

(3) **Reciprocal to distance between points** (Δs):

$$p_{\Delta g} = \frac{c}{\Delta s} \tag{6}$$

(4) **Depending on RMS of mean proportional for gravity difference** ($m_{\Delta g}$) (calculated with SD of readings: $m_{\Delta g}^2 = (SD_j)^2 - (SD_i)^2$):

$$p_{\Delta g} = \frac{c}{m_{\Delta g}^2} \tag{7}$$

(5) **Use of modifying function in robust estimator**.

c. ***Depending on weights for gravimeters***:

Weights for two gravimeters (I and II) could be present with

$$P_I = \frac{c}{\left(m_o^{\Delta g\ I}\right)^2} \text{ and } P_{II} = \frac{c}{\left(m_o^{\Delta g\ II}\right)^2}, \tag{8}$$

then the mean proportional gravity difference from two gravimeters masurements and between points *i* and *j* could be presented with

$$\Delta\bar{g}_{ij} = \frac{\Delta g_{ij}^{I} P_{\Delta g_{ij}}^{I} P_I + \Delta g_{ij}^{II} P_{\Delta g_{ij}}^{II} P_{II}}{P_{\Delta g_{ij}}^{I} P_I + P_{\Delta g_{ij}}^{II} P_{II}}, \tag{9}$$

Where $P_{\Delta g_{ij}}^{I}, P_{\Delta g_{ij}}^{II}$ are weights calculated with formulas given in preceding point **b** (formulas (4) ÷ (7)). Weight of mean proportional gravity difference is

$$P_{\Delta\bar{g}_{ik}} = P_{\Delta g_{ij}}^{I} P_I + P_{\Delta g_{ij}}^{II} P_{II} \tag{10}$$

The final structure of covariance matrix could be formed with use simultaneously (or partly) of listed before approaches **a**, **b** and **c**.

3. PRELIMINARY AND POST-PROCESSING ESTIMATION OF MEASUREMENTS

Preliminary and post-processing of measurements is based on appropriate choice of verifications for statistical hypothesis and appropriate statistical series. The content of these two components depends on main characteristics of relative gravimetric network and realized measurements (listed in point 1.).

Applying of suitable techniques for preliminary and post-processing is ensuring:

(1) **Absence of gross and systematic errors on different stages of processing**

The control for availability of errors is made with use of series of gravity readings, of gravity differences, of closures of figures. Mainly used method for gravimetric networks is τ-test (Pope, 1976). It is applied for residuals, as a result of application could be eliminated measurements with gross errors.

For detecting of outliers are assumed the null and alternative hypothesizes in form

$$\begin{aligned} H_o &: \max\left(\frac{v_i}{\hat{\sigma}_i}\right) < c \\ H_1 &: \max\left(\frac{v_i}{\hat{\sigma}_i}\right) \geq c \end{aligned} . \tag{11}$$

Where $\hat{\sigma}_i$ is RMS of residual, critical value c is calculated with formula

$$c = \tau_{f,1-\alpha_0/2} = \sqrt{\frac{f.t^2{}_{f-1,1-\alpha_0/2}}{f-1+t^2{}_{f-1,1-\alpha_0/2}}}. \tag{12}$$

Student distribution is assigned with t with degrees of freedom f, $\alpha_o = \dfrac{\alpha}{n}$ is local significance level, α is global significance level, which is adopted in range 1% - 5%, $\tau_{f,1-\alpha_0/2}$

is calculated τ distribution.

(2) **Stochastic characteristics of input data (distribution)**

Mainly is supposed that distribution is normal. Standard procedures are used to examine the distribution like χ^2-test (goodness of fit test), ω^2-test and Anderson-Darling test (Boganov and Vuchkov, 1979).

(3) **Determination of a-priory RMS**

Calculation of a-priory RMS values is made upon series in point (1).

(4) **Correct models for drift and calibration**

The control of applied calibration could be made with examine of differences from same closures between used gravimeters $\left(d^w\right)$.

$$d^w = w^I - w^{II}\ ,\ \ p_{d\,w} = \frac{p_{w^I} \cdot p_{w^{II}}}{p_{w^I} + p_{w^{II}}}\ , \tag{13}$$

where

$$\begin{aligned} &w^I, w^{II} &&- closures\ from\ gravimeter I\ and\ II \\ &p_{w^I}, p_{w^{II}} &&- weights of\ closures \\ &p_{dw} &&- weight of\ difference \end{aligned}$$

The **control for gross errors** is made upon calculated a-priory RMS for unit weight $\left(m_{dw}^{a-priory}\right)$:

$$m_{dw}^{a-priory} = \sqrt{\frac{\left[p_{dw} d^w d^w\right]}{N}}\ ,\ \ d^w < 3 m_{dw}^{a-priory} \sqrt{\frac{1}{p_{dw}}}\ . \tag{14}$$

This series $\left(d^w\right)$ must be examined and for availability of systematic errors. This series could be used when there is sufficient number of closures $(N > 30)$.

The other possible control could be made with check between same gravity differences (d) from used gravimeters. Application is in the same manner like this for closures:

$$d\ \ = \Delta g^I - \Delta g^{II}\ ,\ \ p_d\ \ = \frac{p_{\Delta g^I}\ \cdot p_{\Delta g^{II}}}{p_{\Delta g^I}\ \ + p_{\Delta g^{II}}}, \tag{15}$$

where

$$\begin{aligned} &\Delta g^I, \Delta g^{II} &&- gravity differences\ from\ gravimeter I\ and\ II \\ &p_{\Delta g^I}, p_{\Delta g^{II}} &&- weights of\ gravity differences \\ &p_d &&- weight of\ difference \end{aligned}$$

The control for gross errors in that case could be made upon calculated a-priory RMS for unit weight $\left(m_d^{a-priory}\right)$:

$$m_d^{a-priory} = \sqrt{\frac{[p_d\, d\, d]}{N^d}}, \;\; d \;\; < 3 m_d^{a-priory}\sqrt{\frac{1}{p_d}}. \qquad (16)$$

This series (d) must be examined and for availability of systematic errors. This series usually are sufficient number – bigger than N, but again must be $N^d > 30$.

(5) **Detecting of availability of correlation and autocorrelation between errors and time**

This check usually is made for series of residuals. Appropriate criterion for check for availability of autocorrelation is Durbin-Watson test (Kutner M. et al., 2005). Availability of correlation could be made with check for significance of correlation coefficient. The researched hypotheses are:

$$\begin{aligned} &H_o : \rho = 0 \\ &H_a : \rho \neq 0 \end{aligned}, \qquad (17)$$

where

$$\rho - regression coefficient$$

The test statistics is (Kutner M. et al., 2005)

$$t^* = \frac{r\sqrt{n-1}}{\sqrt{1-r^2}}, \qquad (18)$$

where

$$r - Pierson\ correlation\ coefficient$$
$$n - number\ of\ measurements$$

If the next condition is satisfied, then the null hypothesis is accepted.

$$\left|t^*\right| \leq t(1-\alpha/2, n-2), \qquad (19)$$

where

$$\alpha - significance\ level$$
$$t - parameter\ of\ Student\ distribution$$

(6) **Correct and complete model (by global test)**

This check is made with global or χ^2-test of residuals (Walpole and Mayers, 1989 and Caspary, 2000). For the realization of the test is calculated chi-square statistics:

$$\chi^2 = \frac{f.\sigma^2}{\sigma_o^2}, \qquad (20)$$

The critical interval is determined by the inequalities:

$$\chi^2 < \chi^2_{\alpha/2} \text{ and } \chi^2 > \chi^2_{1-\alpha/2}. \tag{21}$$

With σ_o is assigned a priory RMS of unit weight, σ is RMS for unit weight after adjustment, χ^2 is Chi-square distribution with significance level $\alpha/2$ and $1-\alpha/2$, α is global significance level, f are degrees of freedom in adjustment.
If χ^2-test is not fulfilled, indicates for possible availability of: gross errors in measurements, incorrect stochastic model (correlation in measurements) and incorrect functional model.

(7) **Significance of parameters and model adequacy**.
One of the appropriate methods for this examination is presented by (Sevilla et.al., 1990). The method is based on stepwise regression analysis through residuals.

4. ASSESSMENT OF OPTIMAL METHOD FOR ESTIMATION

The choice of evaluation method depends on:

- Type of measurements - complexity of factors influencing the measurements and determining their accuracy;
- Direct or indirect method type of measurements;
- Assurance that the mathematical model presented sufficiently and accurate precise measurements.

Typical for the relative gravimetric measurements is that they are indirect measurements. Factors (internal and external) influencing relative gravimetric measurements are complex, various and difficult for modelling. The influence of these factors is reflecting like anomalous drift, gross and systematic errors. The reasons for them could be availability of disturbances (shocks and vibrations), external disturbances (atmospheric changes, humidity, etc.), internal disturbances (mechanical hysteresis, elastic relaxation). That is the reason to expect not only normal distribution of errors but availability of additional disturbing (contaminate) distribution for part of errors. Appropriate estimation methods in this case are **robust estimation methods**. They are taking into account that the measurement errors may not be only with normal distribution. The robust methods are presenting distribution of errors like sum of main part $(1-\varepsilon)$ with base distribution F and complement part ε with so-called disturbed or contaminated distribution H:

$$G = (1-\varepsilon)F + \varepsilon H. \tag{22}$$

For gravimetric network estimation robust methods are used in establishment of Hungarian Base Network (MGH- 2000) (Csapo et.al., 2003) and in Local Gravity Net in the Province of Valencia (Spain) (Martín et.al., 2011). In this paper are presented results from examination and application of robust method in estimation of Base Gravimetric Network of Macedonia.

For **Local Gravity Net in the Province of Valencia** (Spain) is used robust estimation method based on Huber's robust estimation method. As a result, the weights of measurements are multiplied with factor *k*:

$$
\begin{aligned}
&- \quad k = 1 \qquad \text{if} \qquad |v_{ij}| < \mathrm{k}\sigma; \\
&- \quad k = \frac{\mathrm{k}\sigma}{|v_{\mathrm{ij}}|} \qquad \text{if} \qquad |v_{ij}| < \mathrm{k}\sigma.
\end{aligned} \tag{23}
$$

With σ is denoted RMS, which is calculated for each gravity difference and each gravimeter separately, v_{ij} is the residual for gravity difference.

For **Hungarian Base Network** (MGH- 2000) are used two approaches for adjustment: first is based on condition $\sum |v| = \min$, and second is using Least-Squares Estimation (LSE) method combined with Danish method. For final estimation is used second approach, where weights are determined iteratively by formula

$$
p_{i,j} = \frac{1}{1 + a_k v_{j-1}^2}, \tag{24}
$$

Where v_{j-1} is the residual in $j-1$ iteration, i is the measurement which factor is calculated. Coefficient a_k is calculated with value for v_k with formulas (25), μ_0 is RMS for unit weight:

$$
a_k = \frac{3}{v_k^2}, \; where \qquad
\begin{aligned}
v_k &= 3\mu_o \quad \text{if} \quad v_{\max} > 3\mu_o; \\
v_k &= 2\mu_o \quad \text{if} \quad 2\mu_o < v_{\max} < 3\mu_o; \\
v_k &= \mu_o \quad \text{if} \quad \mu_o < v_{\max} < 2\mu_o.
\end{aligned} \tag{25}
$$

For **Base Gravimetric Network of Republic of Macedonia** is applied again Danish method but in way presented in Caspary (2000):

$$
(P_i)_{k+1} = \begin{cases}
(P_i)_k & |(v_i)_k| \le \dfrac{c.\mu_o}{\sqrt{p_{v_i}}} \\[2ex]
(P_i)_k . \exp\left(- \dfrac{|(v_i)_k . \sqrt{p_{v_i}}|}{c.\mu_o} \right) & |(v_i)_k| > \dfrac{c.\mu_o}{\sqrt{p_{v_i}}}
\end{cases} \tag{26}
$$

Where $(P_i)_{k+1}$ is calculated weight of gravity difference in $k+1$ iteration, constant c is set to be 1.5, p_{v_i} is the weight of the residual.

5. ASSESSMENT OF QUALITY OF MATHEMATICAL MODEL AND OPTIMIZATION CRITERIA

In relative gravimetric network usually are formed and analysed many mathematical models and/or are applied different estimation methods that lead to too many variants of realizations for the network. To choose the most qualitied realization there must be preliminary set of regulations for quality of network. The most often used criteria for quality of network are related with trace and determinant of covariance parameters matrix. These global optimization criteria are M-criterion (M) (Peevski and Zlatanov, 1970) and generalized variance (M_W)

(Graferend et al., 1979). They are presented relevantly with next formulas, where K_x is covariance parameters matrix and n is number of parameters:

$$M = \sqrt{\frac{Sp(K_x)}{n}} \text{ and } M_W = \sqrt[2n]{Det(K_x)}\,. \qquad (27)$$

There are and other appropriate criteria in comparison of many variants like criteria given by Mierlo (1982). Two of them are $\overline{\mu}$ and $\overline{\overline{\mu}}$:

$$\overline{\mu} = \frac{1}{n} Tr\left(K_2^{-1} K_1\right) \text{ and } \overline{\overline{\mu}} = \sqrt[n]{Det\left(K_2^{-1} K_1\right)}, \qquad (28)$$

where K_1 is covariance parameters matrix of examined model and K_2 is appropriate criterion matrix.

6. BASIC GRAVIMETRIC NETWORK OF REPUBLIC OF MACEDONIA

Basic Gravimetric Network of the Republic of Macedonia consist form Zero order gravity network (Absolute Gravimetric Network) and First order gravity network. The Absolute gravity network was established in year 2010 and consists of three stations. The First Order Gravity Network of the Republic of Macedonia is established in 2013 (Geotechengineering & Zenit JV, 2013). The Basic Gravimetric Network is defined by 28 points – 3 absolute points and 25 first order gravity points. **Network configuration** is based on uncovering triangles regularly covering the territory of Macedonia (Fig. 1). Triangles are 41 and are formed from 68 connections (gravity differences between points). Gravity connections between two points for defined line are measured in scheme 1-2-1. This **scheme of measurement** is known as difference method or star method. **Gravimetric measurements** are realized with **two gravimeters** models Scinterx CG3+ (Ser.No. 120140052) and Scintrex CG-5 (Ser. No. 73). Measurements are made simultaneously with both gravimeters. In time of gravimetric measurements are registered air pressure and temperature with two instruments (barometers) of type PHB-318.

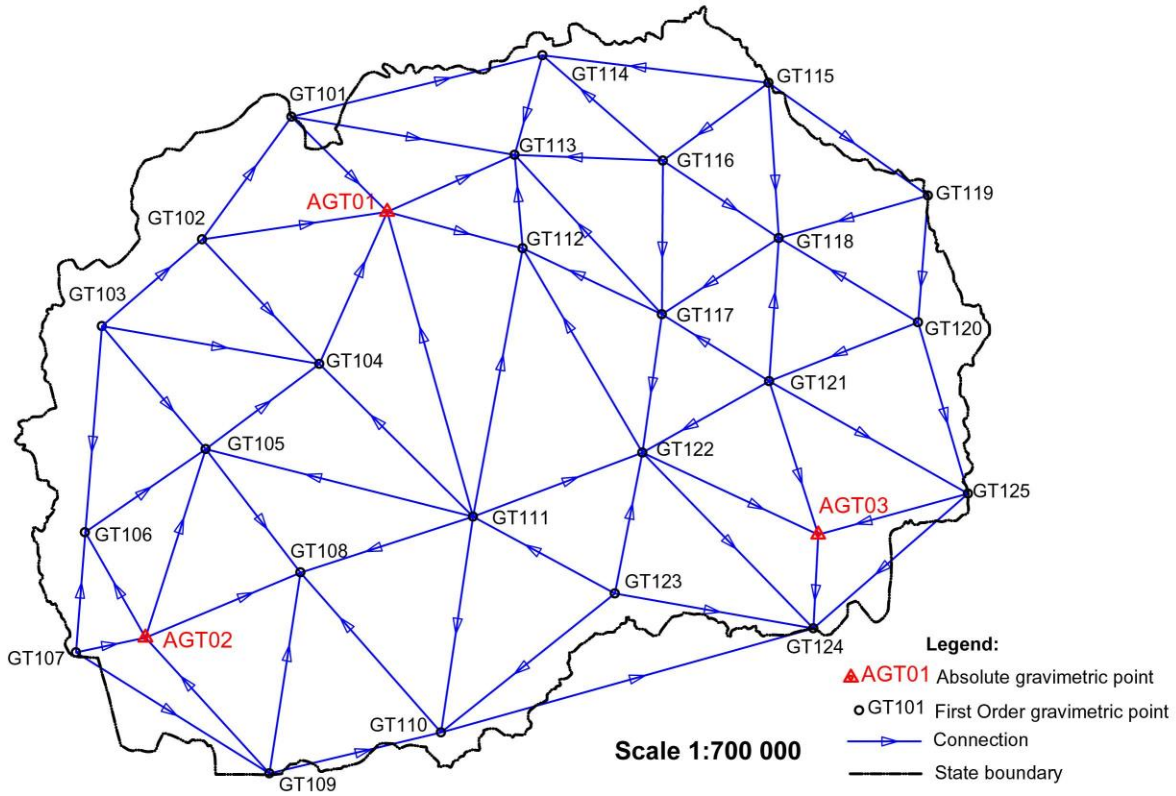

Figure 1. First Order Gravity Network of RM

Used scheme of measurement (1-2-1'-3-1") makes possible determination of drift for each gravity connection separately (d_{12} for 1-2-1' and d_{13} for 1'-3-1"). The drift is calculated over a short measurement time interval, which leads to minimization of its error (it is considered that drift error is proportional to time interval).

$$d_{12} = \frac{O_{1'} - O_1}{t_{1'} - t_1}.\Delta t_{12} \ , \ d_{13} = \frac{O_{1''} - O_{1'}}{t_{1''} - t_{1'}}.\Delta t_{1'3} \tag{29}$$

Calculated gravimetric differences in gravimetric loop has no direct dependence with readings, they are with no common readings. Actually, they are in partly dependence between each other by means of drift.

$$\Delta g_{12} = O_2 - d_{12} - O_1 \ , \ \Delta g_{13} = O_3 - d_{13} - O_{1'} \tag{30}$$

Calculation of gravity difference with closest in time readings leads to limitation of systematic errors on continuation of gravity loop.

Applied approach for measurement and calculation helps to limitation of algebraic and physical correlation between measurements in gravimetric loop. As a result, the studied variants of stochastic models in the adjustment by least squares are represented by diagonal correlation matrix of measured gravimetric differences, i.e. without taking into account the dependency between them. The variants of weights represented by formulas (4), (5) and (7) are investigated. With second variant (with weights reciprocal to time - formula (5)), the time Δt used to determine the weight coincides with the time by which certain drift for the same gravimetric difference is calculated. This is suggesting that connections with drift determined by a longer time interval are less accurate. By so presented stochastic model of measurements, we could say that *errors of the drift determination are a major source of errors in the relative gravimetric measurements*.

In Basic Gravimetric Network of the Republic of Macedonia studied mathematical models are six - three variants of stochastic models applied to two variants of formation of gravimetric differences (by determining the arithmetical mean and proportional mean readings, as a result of the six measured readings in each gravity station).

For measurements with gravimeter CG3+ are formed three arithmetical mean models: CG3_SP1 (equal weights – formula (4)), CG3_SP2 (weights proportional to time – formula (5)) and CG3_SP3 (weights depending on RMS – formula (6)) and three proportional mean models: CG3_TP1 (equal weights – formula (4)), CG3_TP2 (weights proportional to time – formula (5)) and CG3_TP3 (weights depending on RMS – formula (6)).

For measurements with gravimeter CG-5 are formed three arithmetical mean models: CG5_SP1 (equal weights – formula (4)), CG5_SP2 (weights proportional to time – formula (5)) and CG5_SP3 (weights depending on RMS – formula (6)) and three proportional mean models: CG5_TP1 (equal weights – formula (4)), CG5_TP2 (weights proportional to time – formula (5)) and CG5_TP3 (weights depending on RMS – formula (6)).

Here will be presented and compared results from the free gravimetric network adjustment.

For all tested models are applied χ^2-test and τ-test at significance level $\alpha = 0.05$. Critical values for χ^2 are defined by formulas (21) (they are 23.7 and 58.1). The individual values of the criteria of the χ^2- test for each model are shown in **Table 1** for gravimeter CG3 and in **Table 2** for gravimeter CG5. For all models of the two gravimeters global test is satisfied.

The critical value of τ-test, calculated after formula (12) is $\tau_{f,1-\alpha_0/2} = 3.2$. The maximal Studentized residuals max (**v/m$_v$**) for each model are presented in **Table 1** for gravimeter CG3 and in **Table 2** for gravimeter CG5. Tau test is not satisfied for the three arithmetical mean models of gravimeter CG3.

The results obtained by the adjustment of the different models for both gravimeters were compared with criteria: M-criterion (arithmetical mean error) (Peevski and Zlatanov, 1970); generalized variance (geometrical mean error); sum of residuals, multiplied with weights of measurements [pv] and sum of Studentized residuals [v/m$_v$]. The calculated results for gravimeter CG3 are presented in **Table 1** and for gravimeter CG5 in **Table 2**.

For gravimeter CG3 the global accuracy for second models is highest (models CG3_SP2 and CG3_TP2). The third arithmetical mean and proportional mean model (CG3_SP3 and CG3_TP3) has significantly greater value for [pv]. The second proportional mean model can be defined as the most representative (CG3_TP2).

Table 1. Results for χ^2-test, τ - test and global accuracy for free network defined by RMS of values for gravity accelerations of models for gravimeter CG3

model	CG3_SP1	CG3_SP2	CG3_SP3	CG3_TP1	CG3_TP2	CG3_TP3

χ^2 test limits [23.7; 58.1]	37.0	36.3	35.2	47.7	47.5	45.4
τ - test max(v/m_v)	***3.38***	***3.34***	***3.71***	2.30	2.49	-2.27
connection	(118-119)	(118-119)	(118-119)	(114-101)	(117-122)	(113-1)
$M_{\Delta g}$ [μGal]	6.72	6.48	6.64	5.49	**5.23**	5.32
(M_W)$_{\Delta g}$ [μGal]	6.58	6.32	6.40	5.38	**5.10**	5.12
[pv] [μGal]	6.3	3.4	***50.8***	1.7	**2.1**	***-41.3***
[v/m_v]	0.72	0.22	-0.65	0.41	**-0.09**	-1.21

For gravimeter CG5 highest global accuracy is obtained for first and second models (models CG5_SP1, CG5_SP2, CG5_TP1 and CG5_TP2) (**Table 2**). These models can be defined as the most representative.
The third models (CG5_SP3 and CG5_TP3) are again with significantly greater value for [pv]. The results for CG5 gravimeter are with slightly lower global accuracy and with significantly larger values (several times) for [pv]. For both gravimeters, results for the third models are the worst.

Table 2. Results for χ^2-test, τ - test and global accuracy for free network defined by RMS of values for gravity accelerations of models for gravimeter CG5

model	**CG5_SP1**	**CG5_SP2**	**CG5_SP3**	**CG5_TP1**	**CG5_TP2**	**CG5_TP3**
χ^2 test limits [23.7; 58.1]	46.5	47.5	51.4	47.3	48.3	52.2
τ - test max(v/m_v)	2.86	2.31	2.98	2.71	3.10	3.01
connection	(120-118)	(113-1)	(120-118)	(113-1)	(120-118)	(120-118)
$M_{\Delta g}$ [μGal]	6.86	7.06	7.34	6.86	7.06	7.40
(M_W)$_{\Delta g}$ [μGal]	6.72	6.89	7.17	6.72	6.89	7.23
[pv] [μGal]	99.8	34.6	***212.0***	102.0	35.6	***181.5***
[v/m_v]	10.60	9.49	10.72	10.85	9.82	11.41

Results from both gravimeters are processed for combined models and adjustment of free network is presented again. The combined proportional mean models are calculated in two variants – by formulas (9) and (10), with separate weights for each gravimeter (proportional mean models 35m_TP1, 35m_TP2 and 35m_TP3) and without separate weights (proportional mean models 35_TP1, 35_TP2 and 35_TP3). The results from applied global and local tests of residuals and the results from global accuracy of the network are presented in **Table 3**.
Achieved results for the second proportional mean model (35m_TP2) are with highest global accuracy and lowest values for criterions, which shows availability of systematic errors.

Table 3. Results for global accuracy of unknowns for free network for the combined models of measurements with both gravimeters

model	35_TP1	35_TP2	35_TP3	35m_TP1	35m_TP2	35m_TP3
χ^2 test limits [23.7; 58.1]	43.0	44.2	42.4	66.8	44.1	41.5
τ - test max(v/m_v)	-3.07	3.06	-3.04	-2.96	-2.88	-2.85
connection	(120-125)	(120-125)	(120-125)	(120-125)	(120-125)	(120-125)
$M_{\Delta g}$ [μGal]	4.44	4.44	4.65	4.33	**4.24**	4.51
$(M_W)_{\Delta g}$ [μGal]	4.35	4.33	4.52	4.24	**4.14**	4.36
[pv] [μGal]	103.9	37.8	***145.0***	66.8	**21.4**	57.0
[v/m_v]	8.66	7.70	8.84	7.08	5.64	6.00

Variant with estimation of network with application of Danish method is applied. The Danish method is calculated with adopted constant value $c = 2.0$ (formula (26)). In **Table 4** are presented results for tree models based on proportional mean models with applied Danish method (assigned with 35d_TP1, 35d_TP2 and 35d_TP3).

Table 4. Results for global accuracy of unknowns for free network for the combined models of measurements with both gravimeters with applied Danish method

model	35d_TP1	35d_TP2	35d_TP3
χ^2 test limits [23.7; 58.1]	34.4	35.0	38.0
τ - test max(v/m_v)	-1.94	1.90	-1.85
connection	(113-101)	(120-118)	(115-119)
$M_{\Delta g}$ [μGal]	3.15	**3.27**	4.01
$(M_W)_{\Delta g}$ [μGal]	3.08	**3.18**	3.85
[pv] [μGal]	18.6	**4.4**	8.3
[v/m_v]	4.43	**3.63**	3.06

The second analyzed model (35d_TP2) is defined as optimal one. Calculated RMSs for him are a little bit bigger from these calculated for the first model (35d_TP1), but difference in [pv] is significant for the benefit of second model.

7. CONCLUSION

The choice of an appropriate structure of gravimetric network can guarantee the control of the relative gravimetric measurements with preliminary assessment of the accuracy. This provides the necessary security for the absence of gross and systematic errors, compulsory for last squares adjustment.

Especially important in the adjustment of gravimetric networks have realized scheme of measurements. It defines the characteristics and the possibilities to form a mathematical model of the relative gravimetric measurements. The choice of a suitable scheme of measurements can reduce the influence of the algebraic and physical correlation.

The selection of optimum estimation method depends on the possibility of a disturbing distribution of measured quantities. The relative gravimetric measurements are characterized by an indirect method to achieve them, as well as with numerous and difficult to model factors which are influencing them. As a result of the application of robust estimation method - Danish method – are established lower values of RMSs defining global accuracy of the network. The applied method leads to optimal values for [pv] and for the sum of Studentized residuals which are indicated availability of systematic or inadmissible errors. Smaller values for these criteria are obtained by the assumption made for the presence of symmetrical distribution of measurement errors, represented as the sum of base (normal) and contaminated distribution.

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