Deformation Analysis Using L-estimates Method

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Key words：L-estimate; ship-lock deformation; trimming parameter

SUMMARY

As for closing range photogrammetry, it is important to improve the precision of image points when some key factors is considered such as the selection of camera station and photography method as well as the distribution of control points and the procession of image data analysis. L-estimates theory is an effective method to diminish or eliminate measurement error of image points. Combining the practice of ship-lock deformation surveying,test shows the improving effect of image data quality when L-estimates is used under different condition.

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1. INTRODUCTION

Traditionally surveying errors distribution should obedient the Gaussian normal distribution, and its condition to establish statistics is all the surveying data are equal precision, and its distribution centre is effective estimation of mean square error.

However, in some cases, the large surveying errors are found more often than the one of normal distribution, this error is also called gross error which cause the reducing of observations and adjustment result accuracy with the reliability. Such as in the close-range photography, due to the existing of gross error in image observations, cause the accuracy and reliability of exterior orientation elements of the photo with intersection calculation method to be reduced much lower.

In order to diminish or eliminate the impact of such error, robust estimation theory is introduced in the surveying adjustment. In recent years, more than hundred species of robust estimation models have been proposed, which can be summarized in three main categories: L-estimation, M-estimation and R-estimation. And the L-estimation for analysis of the first kind model is been discussed in the paper, Addition with the mention that sample structures are the symmetry in the paper.

2. MATHMATICAL MODEL BUILDING

2.1 Function of Weighted Average Based on L-estimation

L- estimation(Linear Combination of Order Statistics) is based on the thought of arranging a general observation x(n) in order from small to large, or $x_1 < x_2 < \cdots < x_n$,they consist a set of observed values of the order statistics and the arithmetic mean can be seen as a linear combination with the weighting coefficient being 1.

$$T_n = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (1)$$

But more often the case is that considering the order statistic of some functions as (2):

$$T_n = \sum_{i=1}^{n} a_{n_i} \cdot x_i \qquad (2)$$

Here, a_{n_i} is the weighting coefficient of order statistic,if setting h(z)[2] as any functions with characteristics as (2-1) then $a_{n_i} = h(i/n+1)$.

$$h(z) > 0, \quad h(z) = h(1-z), 0 \le z \le 1 \text{ and } \int_0^1 h(z)dz = 0 \qquad (2\text{-}1)$$

Tn is weighted add function with a_{n_i} the order statistic of the weight, i is order number of observation.

For any distribution function *F*, its inverse function usually is defined with quantiles method:

$$F^{-1}(s) = \inf(x|F(x) \geq s)|) \quad 0<s<1 \qquad (3)$$

Obvious： $F(F^{-1}(s)) = S$

As a model for assessing, Gross error sequence can be treated as random or function models. Comparing with observation sequence alignment which contains no gross errors, the former one can be seen as the same mathematical expectation while unusually large variance existed in sequences which contain gross errors; the latter one is seen as the sequence with the same variance and different expectation value.

2.2 α—Trimmed mean

Shown in Figure 1, A set of observations (n) is arranged in size order, form a sequence. Truncated 100α% observations on each end of the sequence, then its averages could be calculated, according to rule of calculus, averages Tn could be acquired in (4):

$$T_n = \frac{1}{(1-2\alpha)n}\int_{\alpha}^{1-\alpha} F^{-1}(s)ds \qquad (4)$$

Here, $s = \frac{i}{n}, \alpha = \frac{h}{n}$, and the means of symbol i，h is same as before.

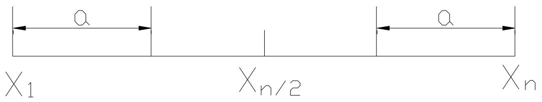

Fig 1 ordering and trimming of observation data

Usually ,（4）could has discrete form as (5):

$$T_\alpha = \frac{1}{(1-2\alpha)n}\sum_{i=\mathrm{int}(\alpha_n)+1}^{n-\mathrm{int}(\alpha_n)} x_i \qquad (5)$$

It can be proved that the variance of α—trimmed average estimation has the characteristics of asymptotic efficiency [3].

For observation, supposing observation error comply with normal distribution, when there are gross errors contained in surveying data,it is inevitable that these surveying data is arranged in the two extremities of the sequence. So the error distribution function should take along with "tail". Choosing α appropriately, the influence of gross error could be eliminated, then the unbiased estimator T_α could be get.

It is obvious that if the value α is taken too large, though the gross errors can be eliminated, but the superiority of assessed parameter also be reduced. So how to find a way to achieve the best combination of the two is discussed below.

2.3 Selection of Combined Parameters Optimization Based on L-estimation

Usually in the close-range photogrammetric, the overlap method of forward intersection and back intersection is used for calculating the object-space coordinates of observation points. where the back intersection is used for acquiring six elements of photo exterior orientation with n control points , its basic error equation is defined as follows:

$$\begin{bmatrix} V_{x_i} \\ V_{y_i} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \end{bmatrix} \begin{bmatrix} \Delta X_S \\ \Delta X_S \\ \Delta X_S \\ \Delta\varphi \\ \Delta\omega \\ \Delta\kappa \end{bmatrix} - \begin{bmatrix} \omega_{x_i} \\ \omega_{z_i} \end{bmatrix} \qquad (6)$$

Using the exterior orientation elements parameter acquired from (6), The forward intersection aims to carry out adjustment calculation with the error equations shown in (7) .

$$\begin{bmatrix} V_{x_i} \\ V_{y_i} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} - \begin{bmatrix} L_{x_i} \\ L_{z_i} \end{bmatrix} \qquad (7)$$

Actually, due to various factors such as the control point impacted by gross error, the solution accuracy could be affected. Thus extracted n_i(at least more than 6) from n control points to be combined. But the data calculated from combination control point are not easy to guarantee that they are unbiased, that is, for the same observation, the data approaching the real value is not in the middle of sequence, but off the center in a way.

Acquiring from 2.1 and 2.2, carrying out the data processing with L-estimation, the first way is using weighted method to acquire weighted average value T_n which can be regarded as parameter estimation as in (8):

$$T_n = \frac{1}{\sum_{i=1}^{n} p_i} \sum_{i=1}^{n} p_i \cdot x_i \qquad (8)$$

The second way is to choose different truncation parameter α for calculating optimal estimation α_0,thus:

$$\alpha = \alpha_0 \quad \forall \quad \frac{1}{(1-2\alpha)^2 n^2} \sum_{i=h}^{n-h} (x_i - T_{\alpha,n})^2 = \min \qquad (9)$$

Supposing that observation in the sequence contains small amount of gross errors, now combining the two estimation methods.

With the first method, observations variance could be estimated. Then the combines of observations variance are arranged in order from small to large, and the observation corresponded the minimum variance should be the most accurate or $\sum p_i v_i^2 = \min$, meanwhile (10)could be acquired:

$$\chi^2(2n_1)_{\min} = \frac{1}{\sigma_0^2} \sum_{i=1}^{2n_1} p_i \cdot v_i^2 \qquad p_i = 1/r_i \qquad (10)$$

Here r_i is redundant observation component, v_i is residuals of observations, pi is Posterior weight of v_i, n_1 is num of combining control points,σ_0 is standard deviation。

The second way, line up the sequence of observations correspond to variance. Set the observed value corresponded to minimum variance as the basis, according to the variance arranged from the smaller to larger, other observations are compared with the base observation individually, the large one place on the right of base observation, and small one on the left. It is seen that the observations order not by size just now, but by the variance value from small to large. After these preparations above finished, taking the minimum variance as basis, carry out *F*-test with (11), setting a certain level of confidence, find out the critical value of maximum variance so as to determine truncation coefficient.

$$F_{\alpha j}(2n_1,2n_1)=\frac{\chi^2(2n_1)_i}{\chi^2(2n_1)_j}=\frac{(\sum_{i=1}^{2n_1}p_i\cdot v_i^2)_i}{(\sum_{i=1}^{2n_1}p_i\cdot v_i^2)_j} \qquad (11)$$

Suppose there are h_1 observations which is greater than $\chi^2(2n)_i$ of critical variance in left tail and h_2 observations in right tail, then the left truncation coefficient α_1 and the right truncation coefficient α_2 are:

$$\alpha_1=\frac{h_1}{n} \qquad (12\text{-}1)$$

$$\alpha_2=\frac{h_2}{n} \qquad (12\text{-}2)$$

The average of estimation parameter of Tn based on α-Trimmed Means is:

$$T_{\alpha_1,\alpha_2,n}=\frac{1}{(1-\alpha_1-\alpha_2)n}\sum_{i=h_1}^{n-h_2}x_i \qquad (13)$$

3. DATA PROCESS OF CLOSING-RANGE PHOTOGRAMMETRY WITH L-ESTIMATION

3.1 Essential Method for Data Process

As mentioned above, L-estimates is used for acquiring adjustment value of unknown parameter obtained from space resection in photogrammetry, nor the direct observation (or residual), so calculated the valuation with L-estimation,these unknown exterior orientation elements are all used as direct observation, and they are independent with the same distribution.

As for the 3 line elements (X_S, Y_S, H_S), there are not any problem for object solution with L- estimation, take their average value as parameter valuations. but for the 3 angle orient elements（φ、ω、κ）,because the rotating matrix obtained from L-estimation cannot meet orthogonal conditions, therefore, it would brings solution errors for the later of space front resection. So carrying out the control point for the various combinations with L-estimators, it is necessary to acquire an orthogonal matrix with Auther Cayley formula.

$$S=(I-A)(I+A)^{-1} \qquad (14)$$

here $$S=\begin{bmatrix}0 & v & u\\ -v & 0 & \lambda\\ -u & -\lambda & 0\end{bmatrix}$$

and $$A=(I-S)(I+S)^{-1} \qquad (15)$$

A is orthogonal matrix,S is real skew-symmetric matrix constructed by three element of（u,v,λ）.

Firstly, calculating the function u_i,v_i,λ_i corresponded to each angle elements in different

S matrix with (6), arranging it in the way of similar exterior orientation angle elements, selecting the best truncation coefficients α_1、α_2 ,find out the averages of three elements S series which is confirmed as final S element, then reverse it with(15) to be amended .acquired the orthogonal rotation matrix R and angle elements （φ、ω、κ）.With the average acquired from 6 exterior orientation elements, space coordinates could be calculated using forward intersection method with (7).

Due to the L-estimation is optimization based on the size variance of the exterior orientation elements, the calculation results can not only eliminate the gross errors of observations, but also improve the impact of solutions which is caused by poor geometric distribution of control point position.

3.2 Result of Data Solution and Analysis

1# shiplock of Gezhou dam is composed of two parts including upper gate and lower gate, test is carried out in lower gate, it includes lock walls and lock gate. UMK-30 and p-31 were used for photographic surveying[6].

42 surveying points were laid out on lock gate, and 21 control points (or check points) were laid on lock wall with 7 photo-camera stations. Coordinates of control point and check point on the lock wall were surveyed by T3 theodolite which can reach precision of ±1mm.

In order to find out the impact of water level to gate, test carried out with four conditions corresponding to water level of 50 m , 55 m, 60 m and 64 m.

In order to verify the feasibility with L-estimation, two water level observation of the highest and the lowest is selected to analysis.

Table 1 is part result calculated by the method of weighted average and α -Trimmed Mean according to the data acquired from two photos based on 07 and 02 camera-station, 9 control has been adopted in the left photo and right photo. The weighted average method adopt $\chi^2(2n_1)_i$ corresponded to minimum variance as criterion, carry out the F-test with the confidence level α=0.0001,

Then the unknown parameters Tn is calculated using (8). For the α-Trimmed Mean, *F*-test is carried out with the confidence level α=0.1,then truncated exterior orientation elements by (12) which is greater than the critical value , further using (13) to obtain the unknown parameter value. Similarly to the photo data derived from 08 and 04 camera station, which also select $\chi^2(2n_1)_i$ as the criterion when adopted two methods as mentioned above.

It can be found in table 1 that the precision of monitored point coordinates calculated by forward intersection is lower than that calculated by regular surveys in the 50m water level, the average difference is 15.6mm. Using the weighted average method, although the accuracy of observation points has been improved 20%, but on the whole, the accuracy is so low that it can not been used for deformation analysis. In contrast, at 64 meters water level, with the same camera(P31)and the same station, using the weighted average method, or α-Trimmed Mean method, the point accuracy calculated by forward intersection is less than 4mm. The reason is because light is not strong and many observation points on image is blur when imaging on the 50m water level. Thus it would bring measurement errors (gross error)for points coordinates, and then lead to the space coordinates of the observation points calculated by forward intersection with larger deviation and lower point accuracy.

Tab1 point result on different water level using method of Weight Mean and α-Trimmed Mean

	Water level:50 米						Water level:64 米					
P-num	Weighted average			α-Trimmed Mean			Weighted average			α-Trimmed Mean		
	X^1	Y^1	H^1	X^2	Y^2	H^2	X^1	Y^1	H^1	X^2	Y^2	H^2
2	12.0	12.1	-10.7	4.7	21.8	3.0	7.0	-0.8	-11.6	2.8	0.7	-3.3
8	9.3	9.7	-9.5	3.9	18.4	-2.2	6.4	-2.2	-9.0	-3.1	-0.3	-3.9
14	8.2	-13.6	-7.5	4.5	21.0	-6.2	4.2	0.5	-4.8	1.6	2.6	1.7
16	18.2	0.5	7.0	16.1	12.3	9.6	5.3	2.8	-2.9	2.9	1.2	-0.6
21	14.1	6.6	0.6	11.5	16.4	-1.4	3.1	0.9	-2.8	0.5	4.8	-2.0
26	6.2	9.1	-8.4	5.0	12.8	-14.6	2.3	2.0	-0.2	0.4	4.0	-2.3
33	12.6	9.2	-0.8	11.0	15.5	-9.4	3.5	2.8	-2.0	1.3	6.5	-4.3
39	13.2	13.6	7.1	11.9	17.8	-4.3	3.3	0.3	0.3	1.1	3.9	13.6
41	13.7	4.9	0.2	13.8	7.3	-6.7	2.2	-2.7	5.0	1.6	0.3	-5.4

Table 2 shows the result of part monitored points acquired from a pair photos taken in left(08) and right(04) camera-station and calculated by ordinary least squares before and after the gross error in control points being eliminated, it has 9 control points in left camera station and 7 in right. It can be seen from Tab.2 that at 50m water level and before the removal of gross errors, due to the gross errors of control points in image, the mean variation of observation points coordinates reach to 70.7mm. But if control point 46 and 47 in right image being removed, it can be seen that .the deviation between monitored point and checked point is reduced significantly, and the point accuracy can reach to 7.5mm. thus it can be diagnosed that the control point 46 and 47 contain gross errors.

Tab 2 compare of point result using LS method in the case of control points containing gross error and eliminating gross error

	Water level:50 米						Water level:64 米					
p-num	gross error not being eliminated			Gross error being eliminated			gross error not being eliminated			Gross error being eliminated		
	X^1	Y^1	H^1	X^2	Y^2	H^2	X^1	Y^1	H^1	X^2	Y^2	H^2
5	72.6	94.8	119.5	1.1	-9.1	-3.4	1.5	-1.8	-2.0	1.5	-1.8	-2.0
9	23.6	30.0	70.0	2.1	-17.0	-6.2	2.8	-10.6	-3.5	2.8	-10.6	-3.5
15	26.8	44.3	56.5	3.2	-11.8	-4.6	3.1	-13.2	-4.2	3.1	-13.2	-4.2
16	47.5	77.4	67.9	2.4	-13.4	-3.6	0.2	-11.6	-3.5	0.2	-11.6	-3.5
21	28.1	50.3	43.7	1.9	-15.1	-2.6	2.0	-11.3	-2.0	2.0	-11.3	-2.0
27	31.9	60.9	28.9	2.5	-14.2	-2.0	2.6	-16.3	-2.2	2.6	-16.3	-2.2
33	35.8	72.5	16.0	2.2	-13.1	-2.1	1.9	-15.7	-0.4	1.9	-15.7	-0.4
39	39.7	79.6	12.5	3.0	-14.8	9.3	1.4	-18.7	0.8	1.4	-18.7	0.8

The result in Tab.3 is calculated with method of the weighted average as well as α-Trimmed Mean in the wake of taking the same control points (gross error not be eliminated) and same image pairs as shown in Tab.2. it can be seen from tab.2, Solution accuracy of monitored points with forward intersection method is improved multiply to ±4mm than the two methods used in Tab.1 and Tab.2

Tab 3 point result using both Weight Mean and α-Trimmed Mean when control point containing gross error

	Water level:50 米		Water level:64 米	
p-	Weighted average	α-Trimmed Mean	Weighted average	α-Trimmed Mean

nu m	X1	Y1	H1	X2	Y2	H2	X1	Y1	H1	X2	Y2	H2
5	0.8	-7.2	-2.7	0.7	-6.8	-2.7	0.8	-2.8	-3.2	0.7	-4.8	-3.5
9	0.3	-10.9	-4.1	0.4	-11.3	-4.2	1.4	-7.5	-3.5	1.6	-9.6	-3.0
15	1.1	-4.4	2.7	1.2	-4.9	-2.8	1.7	-8.0	-4.2	1.7	-9.2	-3.5
16	1.1	-7.7	-2.5	1.1	-7.8	-2.5	-0.5	-7.7	-4.1	-0.6	-8.6	-3.7
21	-0.5	-6.5	-1.1	-0.4	-7.1	-1.2	0.5	-4.0	-2.1	0.3	-4.5	-1.4
27	-0.2	-4.4	-1.2	0.0	-5.1	-1.2	1.0	-6.9	-2.9	0.7	-6.5	-2.2
33	-0.7	-2.4	-1.8	-0.4	-3.2	-1.8	0.3	-4.6	-1.7	0.0	-3.5	-1.1
39	-0.3	-2.8	9.2	-0.1	-3.8	9.4	-0.4	-5.5	-1.5	-0.9	-3.6	-1.0

The data shown in the Tab.3 indicates that adding all control points in least squares adjustment would lead the result poor due to some control points contain gross error or points geometric distribution is poor, Instead when L-estimate method is adopted, in the same conditions, adjustment results is better than least squares results which has eliminated the gross error. Thus, for data processing of close-range photogrammetric, when control point with poor geometry distribution or containing gross errors, α-Trimmed Mean method is superior to least-squares method.

4 CONCLUSION

Through calculations and analysis, we can see that α-Trimmed Mean method is an efficient way of optimum parameter estimation for photogrammetric based on control point backward intersection. The result shows that in the case of part of the control point being poor geometry distribution or containing gross errors, solution with L-estimation is better than the results using least squares.

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BIOGRAPHICAL NOTES

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