DETERMINATION OF GEOID IN THE KINGDOM OF SAUDI ARABIA USING GLOBAL GRAVITY MODEL AND GPS/BENCHMARK DATA: A CASE STUDY.

Ramazan YANAR and Ali ALOMAR, Kingdom of Saudi Arabia.

Key words: Geoid, Mean Sea Level, Surface Fitting, GNSS, Kriging, Orthometric Heights

SUMMARY

A Vertical Reference Frame forms the basis for all developement projects in which heights are used. Heights are generally considered to refer to mean sea level (MSL) and most vertical reference frame attempt to approximate MSL as the datum for heights. In principle the geoid is the ideal datum. In practice both the geoid and MSL are approximated by taking tide gauge measurements at one or more sites over a limited time period.

Nowadays most control survey are established using Global Navigation Satellite System (GNSS). The reference frame for GNSS is the WGS84 where heights are referred to GWS84 ellipsoid, not to MSL. Consequently, in order to reference GPS – derived heights to the geoid, the geoid – ellipsoid separation (N) must be known. Two models for this separation has been implemented using GNSS/Benchmark data for Saudi Arabia.

Vertical reference frame of Saudi Arabia, established in early 1970's as first order vertical control network by spirit leveling based on tidal gauges along the Red Sea and Arabian Gulf, is analyzed for determination of GPS/Benchmark geoid of the Kingdom.

The analysis is based on existing benchmarks and newly created benchmarks essential to places where there are no benchmarks.

Thin Plate Surface Fitting using Least Squares Collocation and Surface Fitting based on Kriging Algorithm was used to derive the conversion surface throughout Saudi Arabia by differencing ellipsoidal heights and orthometric heights on leveled benchmarks occupied by GPS and geoid undulation of Global Gravity Field Model, EGM96 ($\Delta N = N_{MSL} - N_{EGM96}$)

The accuracy analysis is based on comparisons of both cases of geoid (N_{MSL}) and residual geoid ($\Delta N_{MSL-EGM}$) using two algorithms mentioned above. The RMS of surfaces determined by geoid fitting was found 0.123 and 0.088 meter respectively where as the RMS of differences of surfaces determined by residual geoid fitting was found 0.075 m.

This results may indicates that refinement of undulation of Global Gravity Field Model with GPS/Benchmark data gave the preliminary values of geoid undulations acceptable for practice purposes for The Kingdom.

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1. EXISTING NATIONAL GEODETIC VERTICAL NETWORK OF SAUDI ARABIA

The current National Geodetic Vertical Network (NGVN) in the Saudi Arabia is based mainly upon a series of leveling runs carried out during 1966 - 1970. A total of about, 54 levelling circuits were carried out over a total of consisting of distance of 1952 km of first order leveling and 13002 km of second order levelling and around 2500 banchmark established at an average of 6 km. These benchmarks are generally along the major routes and 50-100 m off the road.

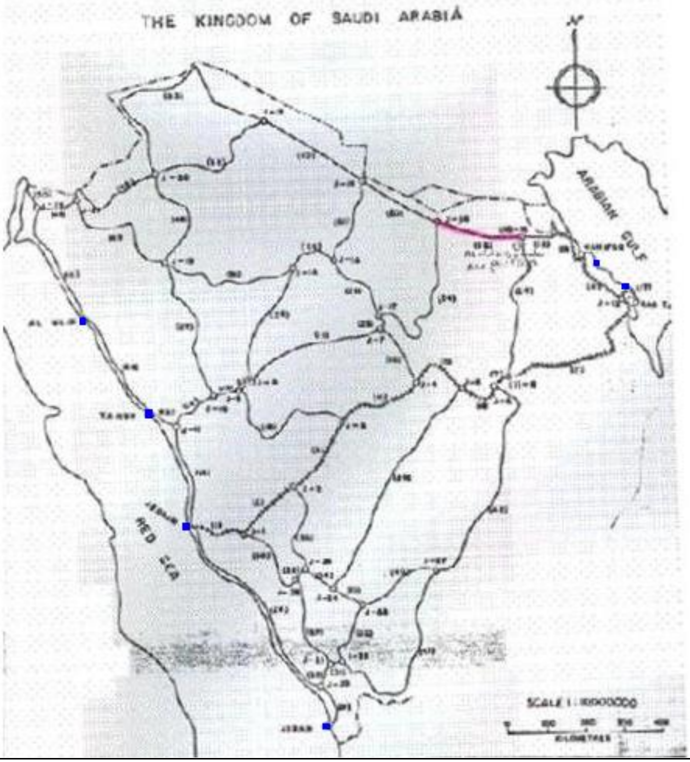

Figure 1.1: National Geodetic Vertical Network of Saudi Arabia

The accuracy was specified as first, second and third order leveling criteria of classification of leveling network. 809 benchmark of the 2500 ground markers, were spirit levelled. The rest were heighted by reciprocal vertical angles.

The vertical datum based on 6 recording tidal observatories, which have data from April 1969 to March 1970. These observatories are located in Jeddah, Yanbu, Al-Wajh, Gizan, Manifah and Ras Tanurah, shown with blue colored square in Figure 1.1. (Nakiboglu, S.M.et all, (1994).

Concrete monuments were installed in all non-rocky and non-sandy locations. In rocky locations where excavation proved difficult a shortened central tube was used. In sandy locations liable to erosion, the three-metre pipe marker was installed with witness posts.

A comprehensive field reconnaissance has been carried out based on the existing banchmarks, height data which is collected, classified and evaluated for GPS surveys. Many of banchmarks of NGVN have subsequently been destroyed and/or lost during development work, and that only as few as 20% has been recovered in some areas (see Figure 1.2).

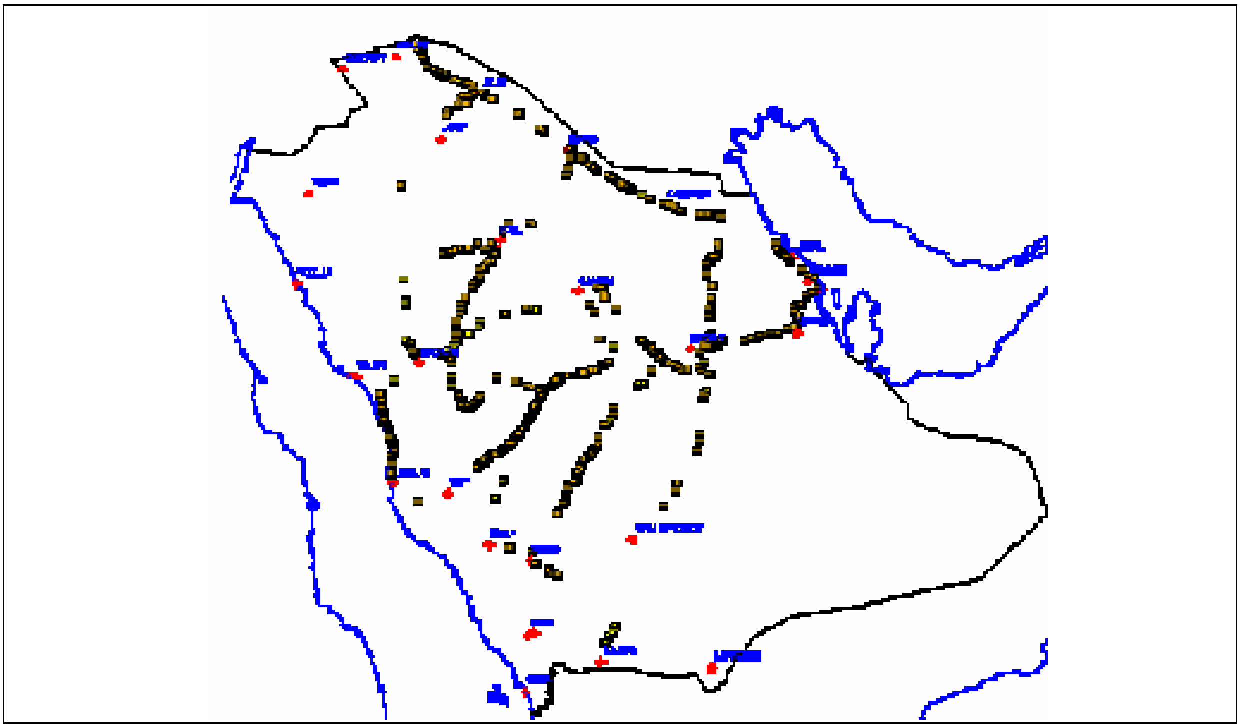

Figure 1.2: Recovered banchmark geometry with horizontal coordinates of NGVN

Additional banchmarks established trought projects by municipalities and other agencies has been included to the process see Figure 1.3.

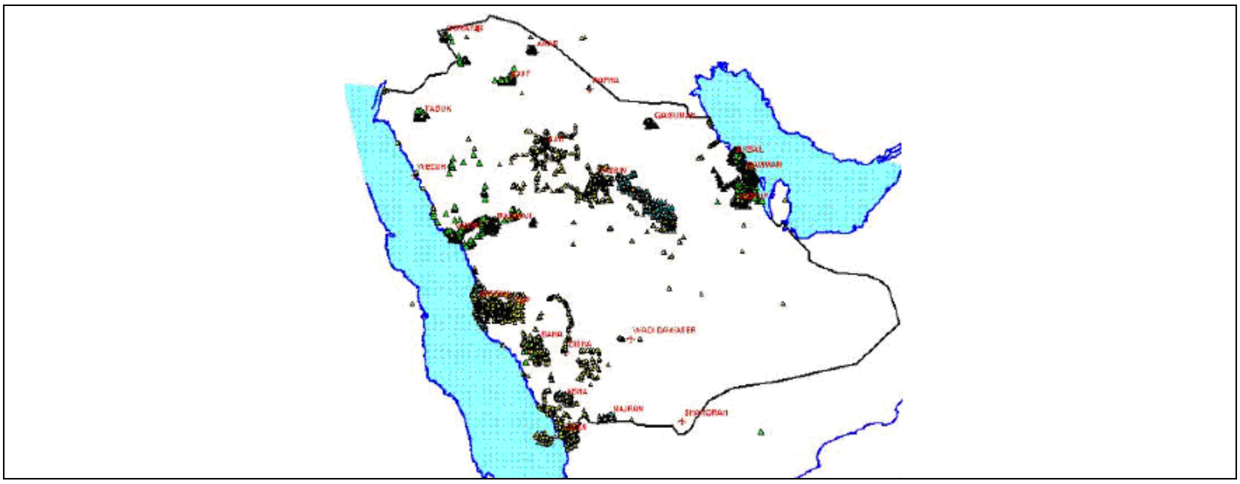

Figure 1.3: Additional banchmark geometry with horizontal coordinates

By examining the existing benchmarks it was decided to carry out spirit leveling in mostely needed areas where existing benchmarks are inadequate and the geoid is steep. In this context we selected South-West and South of Saudi Arabia for spirit leveling. The total distance of monumented lines of Spirit Leveling works was around 2000 km, where benchmarks are constructed at 5-km intervals and named in a way to be consistent with the existing leveling network. As a result, the total number of benchmark used to refine The Earth Geo-potential Model EGM96 is about 3800 points see Figure 1.4.

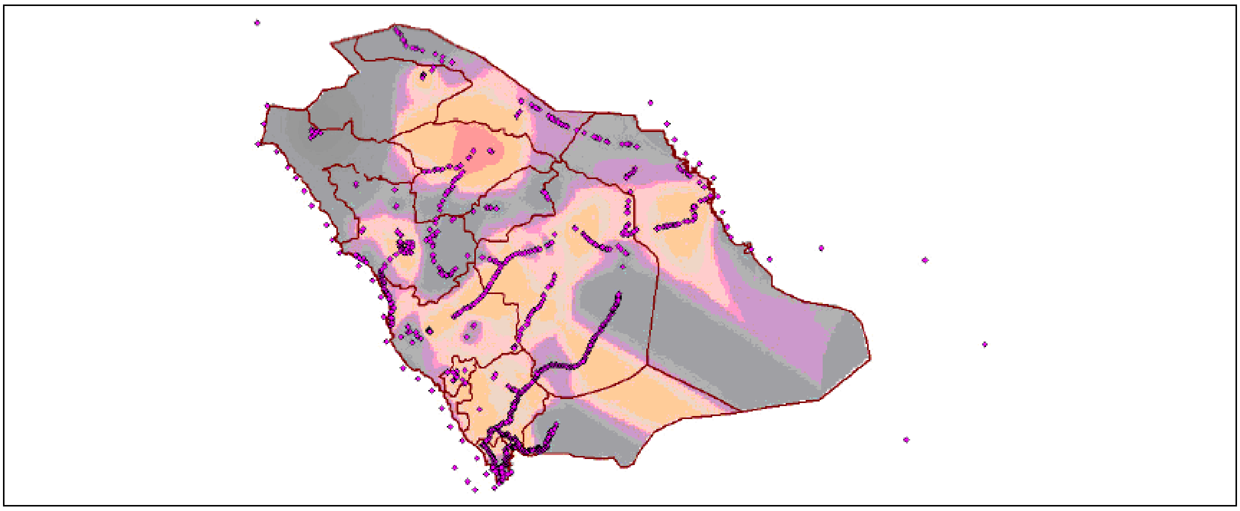

Figure 1.4: All points used to test gridding algoritms

Vertical control analysis showed that the existing leveling network was adjusted to a mean sea level value derived from the tidal observation at Jeddah, Yanbu, Al Wajh, Jizan, Manifah and Ras Tanurah. But, unfortunately, the uncertainties of benchmarks elevations are not available. The only estimation regarding the reliability of benchmarks may be deduced from the difference in elevation obtained from opposite runs shall not be greater than, first and second order leveling criteria of classification of leveling network. Therefore it has been followed this concept during the work.

2. RELATIONS BETWEEN ORTHOMETRIC HEIGHTS AND GEOID

The GPS measured heights are measured from the ellipsoid; therefore, they need to be converted into an orthometric height system. The current methods of converting GPS elevations to orthometric elevations (Martin, Daniel J. (2001) are:

- To incorporate a priori geoid undulation data in three-dimensional (3D) adjustment which holding the benchmark elevations fixed for stations with known values determined by spirit leveling. The minimum number of benchmarks should be four, well distributed through out the region.
- Determination of orthometric heights from GPS vector baseline data involves performing 3D adjustment without using geoid undulation data. In this method the bench marks elevations are held fixed while using zero values for geoid undulations in a 3D adjustment. This interpolates the geoid undulation values for the rest of the stations in the region. Here also minimums of four known benchmarks are needed and preferably more than four well distributed to achieve valid results.
- The best method is to compute the actual geoid undulation difference details from gravity anomalies for the desired stations where ever centimeter accuracies are derived.

The ultimate aim for the vertical network is to determine a geoid surface across the region, in such a way that GPS observations can be corrected so that they agree with the orthometric height datum.

An initial assessment of problems associated with the geoid was made using the observed WGS-84 coordinates with GPS points across the region. From the differences between GPS and orthometric height, the initial values for the geoid separation, N_{MSL} (see Figure 2.1) was determined.

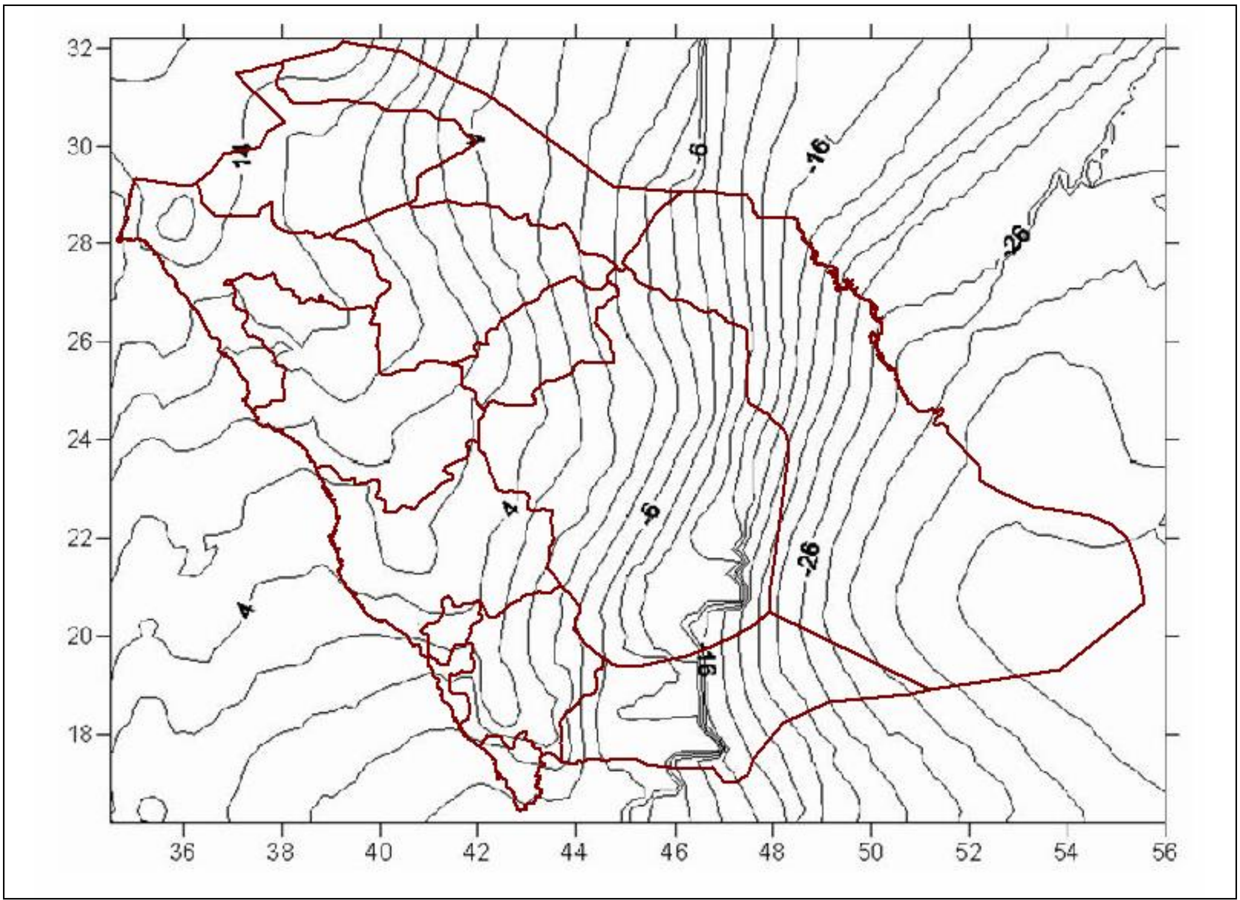

Figure 2.1: WGS84 geoid of The Kingdom

This figure was then compared with the value given by the N_{EGM96} global Earth model. The statistical results are shown in Table 2.1.

Table 2.1 Statistical result of base points of comparison between WGS84 and EGM96 geoids

	Observed geoid minus EGM96 residual variations for base points
Min [m]	-3.376
Max [m]	2.490
Average [m]	0.148
RMS [m]	0.749

RMS of residuals variations is 0.749 m. However, considering the known accuracy of EGM96, the type of terrain, and the area covered, these variations are far greater than would be expected.

3. GLOBAL GRAVITY FIELD MODEL, EGM96

The global Earth Gravity Model for the year 1996 (EGM96) is the result of collaboration between the National Imagery and Mapping Agency, the NASA Goddard Space Flight Centre, and the Ohio State University and covering the Earth gravity field up to degree and order 360. This corresponds to a spatial resolution of up to 55 km and models the geoid within an accuracy of about 40 cm (global average).

For a further refinement of the EGM96 in the Saudi Arabia, surface point information has to be introduced. A point separation of less than 30 km is needed to cover the short wave length part of the harmonic development of the Earth gravity field (degree 361 . . . 10000, 0.5 m . . . 0.01 m geoid undulation, point separation 30 km . . . 1 km (grid wise)).

Figure 3.1 shows the part of the EGM96 model that covers the Saudi Arabia. Proceeding along the 19-degree latitude circle from west to east (from Baha to Sharurah, marked with the circle in red color), a steep increase of the geoidal height of about 5 m (from 3 m to -2 m) occurs over a short distance of less than 100 km.

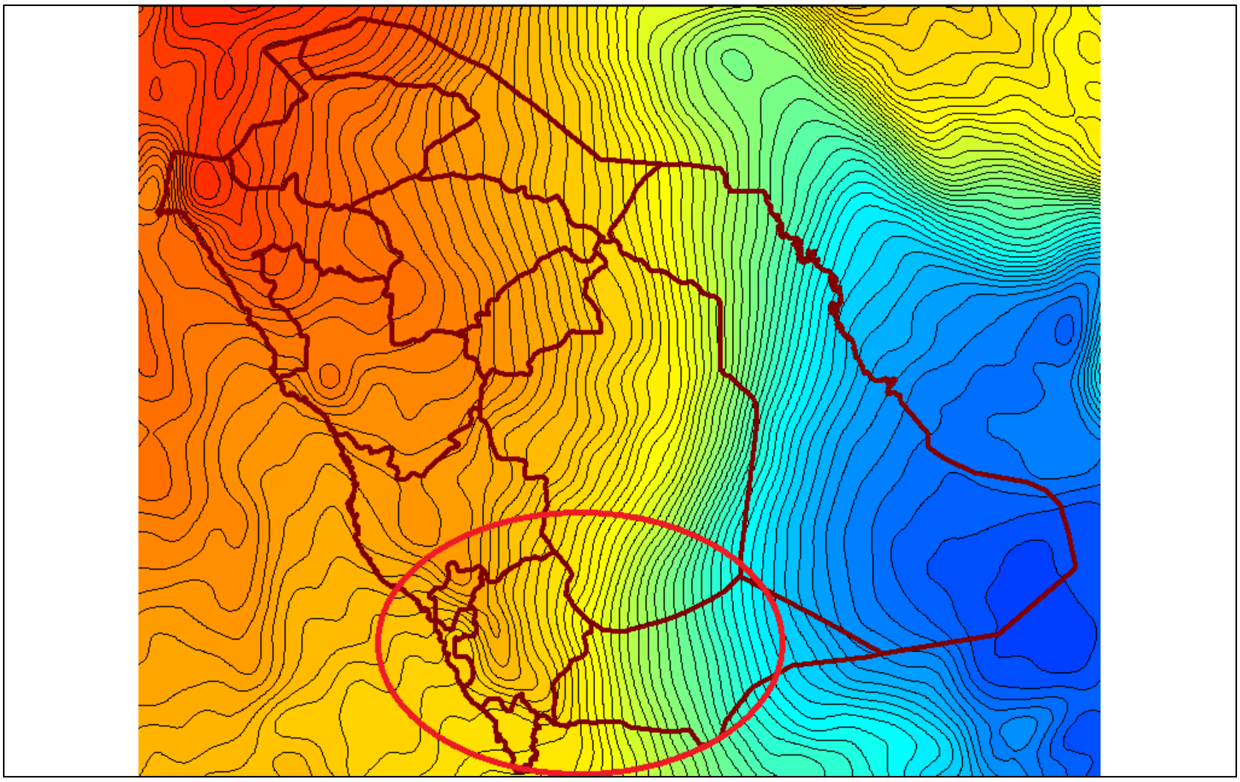

Figure 3.1: EGM96 Geoid Model for the Saudi Arabia

4. DETERMINATION OF GEOID THROUGHOUT THE KINGDOM

In order to determine the geoid throughout the Kingdom, recovered benchmarks (BM) from existing vertical network, selected constraining points on water surfaces around The Kingdom, newly-established benchmarks (GPS-BM) where the geoid steep and benchmark established through projects by different agencies have been used. The orthometric heights of all the points on ground are known from Leveling activities. With ellipsoidal heights coming from global navigation satellite system (GNSS), we have the geoid heights for all these points from the famous relation (4.1).

Furthermore, the points on water surfaces have known orthometric heights of zero. Since these points are used exclusively for constraining purposes, it is thought to be sufficient to take their geoid heights from EGM96. This gives us enough data to estimate the ellipsoidal height at these points, if needed.

Both sides of Equation (4.1) may be arrived at by separate means. EGM96 model can give one estimate of geoid height (N_{EGM96}) and removing the orthometric height from an ellipsoidal height (N_{MSL}, geoid as derived from actual survey data) will yield another. The residual difference between these two estimates and N_{MSL} was processed to develop a model of the correlated signal according to:

$$h_{GPS} - H_{BM} = N_{GPSBM} = N_{MSL} \tag{4.1}$$

$$h_{GPS} - H_{BM} - N_{EGM\,96} = \Delta N_{GPSBM-EGM} = \boldsymbol{\Delta N}_{MSL\text{-}EGM} \tag{4.2}$$

Where: *H:* is the orthometric height; *h:* is the ellipsoidal height; *N:* is the geoid height.

This process involved determining a conversion surface that approximated the correlated signal existing in GNSS-BM residuals ($\Delta N_{EGM\text{-}MSL}$) and N_{MSL}. In this context we tested the following algorithms with the powerful software packages "Golden Surfer 8" and home made software for the determination of the analytical geoid and residual geoid surface (Golden Software,Inc, Briggs, I. C. (1974), Nakagawa, H. et all (2003), Ghilani, Charles D. et all. (2002), Journel, A. G. et all. (1978), Kitanidis, P. K. (1997), Martensson, S. (2002)).

- Bi-Cubic Spline;
- Inverse Distance to a Power;
- Kriging;
- Least Squares Collocation;
- Modified Shepard's;
- Moving Average;
- Polynomial Regression;
- Trigonometric Function; and
- Thin-Plate Smoothing Spline based on Collocation Matrix

Initial tests revealed better results of 2 algorithms when compared to others, for GNSS-BM residuals ($\Delta N_{EGM\text{-}MSL}$) and N_{MSL}. These were: Kriging and Thin-Plate. Thus, we continue herein with these algorithms only.

Each algorithm was used to fit a surface to the available points in hand. Then that surface was used to interpolate for the heights at the same points (base points) and at the points which are not used to fit a surface (rover points), we had on land. This interpolation was done using Biharmonic Spline algorithm (Sandwell, D. T. (1987).

Since we already know the heights of those points, and since we are using unified interpolation algorithm, then we could judge which of the two gridding techniques give better results for our specific test case.

All this was done twice: once with the geoid (N_{MSL}) heights of the points themselves, and once with the differences between these heights and those acquired from the global Earth Gravity Model for the year 1996 (EGM96_N), shown in Figure 3.1 for Saudi Arabia. The second methodology allows us to get rid of the trend and make benefit out of the precision of EGM96.

4.1 Thin Plate Surface Fitting using Least Squares Collocation Matrix Algorithm

This mathematical algorithm can be efficiently applied to geostatistical problems, where a thin-plate smoothing spline (f) is implemented such that it is the unique minimizer of the weighted sum

P*E(f) + (1-P)*R(f) (4.3)

with E(f) the error measure

E(f) = sum_j { | Y(:,j) - f(X(:,j)) |^2 : j=1,...,n } (4.4)

and R(f) the roughness measure

R(f) = integral (D_1 D_1 f)^2 + 2(D_1 D_2 f)^2 + (D_2 D_2 f)^2 (4.5)

Here, the integral is taken over the entire 2-space, and (D_i) denotes differentiation with respect to the (i-th) argument, hence the integral involves the second derivatives of (f). The smoothing parameter (P) is chosen in an (ad hoc) fashion in dependence on the sites X.

In other words, thin-plate spline approximations (f) can be created such that they satisfy, approximately or exactly, the equation for given data values (z) for z = f(x,y), at given scattered data sites (x, y) in the plane. The associated collocation matrix is provided implicitly. The spline created is in stform, as are its first-order derivatives.

Within the calculations, thin-plate smoothing spline builds and uses collocation matrix for scattered data, implicitly. This makes this algorithm powerful (Golden Software,Inc).

Thin Plate algorithm was used to fit a surface to the geoid (N_{MSL}) and residual geoid ($\Delta N_{MSL-EGM}$) in hand separately. Then those surfaces were used to interpolate for the geoid heights (N_{INTP}) and residual geoid heights (ΔN_{INTP}) at the same base points and at the rover points, we had on land.

The RMS of differences between geoid height and interpolated geoid height of base and rover points, residual geoid height and interpolated residual geoid height of base and rover points, has been found 0.142 m, 0.264 m, 0.120 m, and 0.266 m respectively see Table 4.1. The fitted surfaces are shown in Figure 4.1 and Figure 4.2 below.

Table 4.1 the statistics of surfaces determined by geoid (N_{MSL}) and residual geoid ($\Delta N_{MSL-EGM}$) fitting using thin plate algorithm.

	Thin-Plate with Geoid (N_{MSL}) and Residual Geoid ($\Delta N_{MSL-EGM}$)			
	Geoid (N_{MSL})		**Residual Geoid ($\Delta N_{MSL-EGM}$)**	
	Base points	**Rover points**	**Base points**	**Rover points**
Min [m]	-1.767	-0.811	-0.886	-0.780
Max [m]	1.351	0.831	1.204	0.929
Average [m]	0.000	-0.053	-0.001	-0.034
RMS [m]	0.142	0.264	0.120	0.266

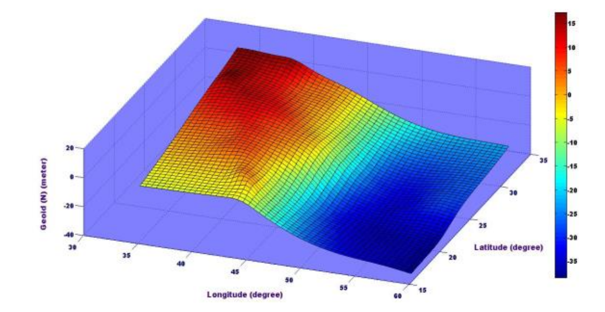

Figure 4.1: Fitted surface to geoid (N_{MSL}) base points values using Thin-Plate

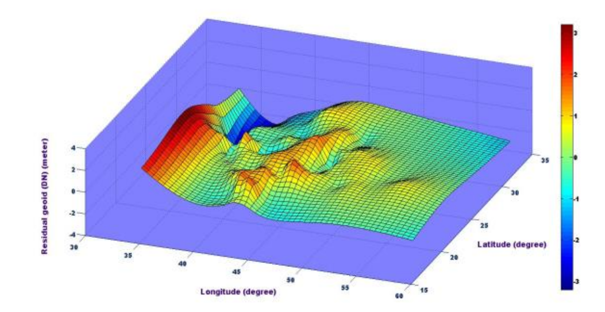

Figure 4.2: Fitted surface to residual geoid ($\Delta N_{MSL-EGM}$) base points values using Thin-Plate

This geometrical elevation grid is used in computing the residuals of the geoid heights computed from the EGM96, and adding appropriate corrections to the geoid heights of

EGM96 according to the relationship shown in (4.2).

4.2 Surface Fitting based on Kriging Algorithm

This geostatistical algorithm has been used worldwide in various similar cases; for example. Kriging is a technique that provides the Best Linear Unbiased Estimator of the unknown fields.It is a local estimator that can provide the interpolation and extrapolation of the originally sparsely sampled data that are assumed to be reasonably characterized by the Intrinsic Statistical Model (ISM). An ISM does not require the quantity of interest to be stationary, i.e. its mean and standard deviation are independent of position, but rather its covariance function depends on the separation of two data points only (Golden Software,Inc), Kitanidis, P. K. (1997), i.e.

$$E\,[(z(x) - m)(z(x') - m)] = C(h) \tag{4.6}$$

where m is the mean of z(x) and $C(h)$ is the covariance function with lag h, with h being the distance between two samples x and x':

$$h = \| x - x' \| = \sqrt{(x_1 - x'_1)^2 + (x_2 - x'_2)^2 + (x_3 - x'_3)^2}. \tag{4.7}$$

While the RMS of differences between geoid height and interpolated geoid height of base and rover points, residual geoid height and interpolated residual geoid height of base and rover points, has been found 0.116 m, 0.365 m, 0.088 m, and 0.240 m respectively see Table 4.2. The fitted surfaces are shown in Figure 4.3 and Figure 4.4 below.

Table 4.2: The statistics of surfaces determined by geoid (N_{MSL}) and residual geoid ($\Delta N_{MSL-EGM}$) fitting using kriging algorithm

	Kriging with Geoid (N_{MSL}) and Residual Geoid ($\Delta N_{MSL-EGM}$)			
	Geoid (N_{MSL})		**Residual Geoid ($\Delta N_{MSL-EGM}$)**	
	Base points	**Rover points**	**Base points**	**Rover points**
Min [m]	-1.063	-2.582	-0.648	-0.905
Max [m]	3.079	0.994	0.848	0.744
Average [m]	-0.002	-0.078	0.000	-0.040
RMS [m]	0.116	0.365	0.088	0.240

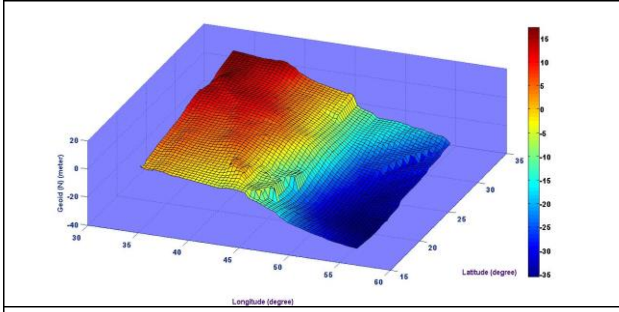

Figure 4.3: Fitted surface to geoid (N_{MSL}) base points values using Kriging

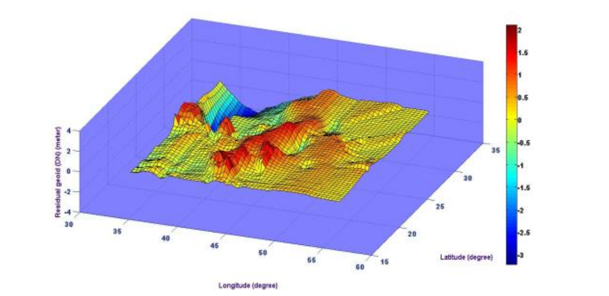

Figure 4.4: Fitted surface to residual geoid ($\Delta N_{MSL-EGM}$) base points values using Kriging

5. RESULTS AND DISCUSSIONS

All these analytical surfaces can be considered to compute the residual geoid heights with respect to the EGM96 surface throughout Saudi Arabia. Then, these corrections can be added to the corresponding EGM96 geoid heights to obtain improved geoid heights at these points.

In order to estimate the accuracy, comparisons have been made for both cases of geoid and residual geoid using two algorithms mentioned above. The statistical results are seen in Table 4.1, Table 4.2 and Table 5.1 for geoid and residual geoid fitting for both algorithms respectively.

Table 5.1: The statistics of differences surfaces determined by geoid (N_{MSL}) and residual geoid fitting (ΔN_{MSL}) using both thin plate and kriging algorithms.

	Differences of Thin-Plate and Kriging			
	Geoid (N_{MSL})		**Residual Geoid ($\Delta N_{MSL-EGM}$)**	
	Base points	**Rover points**	**Base points**	**Rover points**
Min [m]	-1.101	-2.125	-0.945	-0.470
Max [m]	2.967	0.786	0.652	0.414
Average [m]	-0.003	-0.025	0.001	-0.006
RMS [m]	0.118	0.265	0.075	0.106

It is reasonable to interpret the results as geoid precision. These discrepancies are due to error sources from GPS and leveling surveys, mathematical model used as well as from the EGM96 itself.

The results in Table 4.2 indicate that Kriging has yielded the best fitting analytical surface for residual geoid fitting. Therefore, this algorithm was used for the computation of conversion surface of Saudi Arabia.

The geoid conversion surface is estimated to yield decimeter level precision geoid height throughout the Kingdom. Of course, the precision varies depending on the proximity to GPS leveled benchmarks. It is higher in the vicinity of such points and gets lower as the distance gets higher.

It is to be pointed out that the geoid conversion surface of Saudi Arabia input data is not error free. It can be concluded that both ellipsoidal heights determined by GPS techniques and orthometric heights determined by geometric leveling have precision in centimeter level.

The RMS of differences between residual geoid height and interpolated residual geoid height of base points and The RMS of differences of two algorithms has been found 0.088 m and 0.075 m respectively; This results may indicates that refinement of spherical harmonic geopotential model for the year 1996 with GPS/Benchmark data gave the preliminary values of geoid undulations acceptable for practice purposes.

On the other hand, much of the current vertical control was lost due to the destruction or disruption, and because the network was not dense enough to support the use of GPS to get elevations, a precise geoid model should be determined based on a combination of Gravity with GPS / leveling.

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BIOGRAPHICAL NOTES

Ramazan YANAR completed a B.Sc. and PhD in Geodesy from Yildiz Technical University, Istanbul, on the *Geodetic Data Base Design* and *Height System*, his PhD thesis were focused on *Geodetic Height Systems*. He is now in charge of the Surveying and Mapping Department, Ministry of Municipal and Rural Affairs, Riyadh; where he also teaches principles of the Global Navigation Satellite System. His main scientific interests in 3D modeling and establishment of Geodetic Networks (Gravity, CORS, Horizontal and Vertical Ground Control, GPS/Leveling).

Eng. Ali S.Al OMAR completed a B. Sc and M.Sc in Real Estate Development from King Saud University, Riyadh, on *the Surveying and Real Estate*, his M.Sc thesis were focused on *Real Estate Development and City Planning*. He is Incharge of the Directorate for Cadastral Surveying and Mapping in Ministry of Municipal and Rural Affairs, Riyadh. He is managing the 3D modeling, Geodetic, Land, Cadastral and Aerial Survey, Geodetic Networks, CORS, DTM and GPS/Leveling.

CONTACTS

Ramazan YANAR
Ministry of Municipal and Rural Affairs
General Directorate for Surveying and Mapping
PO BOX 53179
Riyadh, Kingdom of Saudi Arabia
Tel: Office 456999 Ext. 3115
Direct: 4531703
Mobile. + 966533273250
Email: *ryanar2012@gmail.com*

Eng. Ali ALOMAR
Ministry of Municipal and Rural Affairs
General Directorate for Surveying and Mapping
P.O Box 291201
Riyadh 11362
Kingdom of Saudi Arabia
Tel: Office 456999 Ext. 3115
Direct: 4531703
Mobile: 0504408043
Email: *ali.s.alomar@gmail.com*