Elastic Conformal Transformation of Digital Images

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SUMMARY

A novel transformation model for registration of geometrically distorted digital images is proposed in the contribution. The registration method is based on a set of ground control points (GCPs) whose coordinates have been captured with limited accuracy. The spatial inaccuracy of the GCPs influences precision of transformation between input and reference images. Quality of the transformation is also affected by unknown nonlinear elastic distortions of the input image. Simultaneous impact of the both sources of inaccuracy results in spatial imprecision of the transformed image. Correct estimation of the resulting spatial imprecision is significant constituent of the proposed registration method. Theoretical principle of the proposed method stems from theory of Gaussian processes (collocation, kriging) and is worked out with the aid of Bayesian approach. The overall solution embodies advantages of non-parametric and parametric estimation - it is both data-driven and tunable by a simple set of parameters.

The proposed method of image registration was implemented as a web application by means of up-to-date software standards of Internet technology. The application is freely available at http://www.vugtk.cz/igc/apps/transformation/ for any Internet user. The proposed registration method can be easily applied in many areas of geodesy and cartography, and remote sensing, e.g. matching maps, georeferencing of satellite or aerial images, spatial data quality management in GIS, cadastre surveying, deformation modeling etc.

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1. INTRODUCTION

Coordinate transformation is frequent task in geodesy and cartography, namely when a GIS is created or updated by a composition of digital images. Such digital images can originate from miscellaneous sources, e.g. aerial or satellite cameras, digitized analogue maps, infrared cameras, radar scenes etc. One important technique to compose different digital images is image registration. Application width of image registration spreads out widely over the branch of geodesy and cartography. It has been extended to a number of other branches, namely medical imaging, robot vision, microscopy, video and multimedia processing, deformation analysis etc. All the applications of image registration can be divided in two main classes: change detection and mosaicing. The both application areas are very promising today.

Comprehensive overview of the image registration methods offers [5]. Transformation models which have been mostly used in image registration are usually set up by parametric estimation. Typical example of such a parametric transformation model is affine, polynomial, perspective or spline transformation. These transformation models are easy to implement, but accuracy analysis of resulting registration can be misleading when the chosen transformation model is corrupted by some unknown irregular factors. This disadvantage can be dissolved by non-parametric estimation methods which are based rather on measured data than on some artificial presumptions as polynomial approximation. Usage of non-parametric methods is not so straightforward and therefore less popular. Furthermore, computational demands of non-parametric methods are higher. The proposed method of image registration has advantageous features of the both approaches. It is tunable by an explicit set of geometrical and statistical parameters. Simultaneously, it is also data-driven since the statistical parameters allow feasible matching of the transformation model to the measured data. The method stems from collocation method. Statistical properties of collocation (see [2]) are emphasized in this contribution. Bayesian approach [1] is applied to estimation of the statistical parameters.

2. PROBLEM FORMULATION

2.1 Required result

Transformation procedure between two digital images has to be designed. The required transformation has to coincide approximately at some ground control points (GCPs). The coincidence has to be as tight as precise the ground control points are. The required transformation need not be strictly linear (i.e. slight elastic distortions are allowed), but has to be conformal. Spatial accuracy of any transformed point has to be estimated as well.

2.2 Given assumptions

Both given images have their own coordinate systems. Coordinates of points in the input image are called *input coordinates*, coordinates of points in the reference image are called *reference coordinates*. A region of interest is given in the overlapping area of the given images. The required transformation can be expressed as mapping

$$\mathbf{t} : \mathbb{R}^2 \to \mathbb{R}^2 : [x, y] \mapsto \mathbf{t}(x, y) = [X, Y] ,$$

where

x, y … input coordinates,

X, Y … reference coordinates.

Approximate similarity transform holds between both coordinate systems in the given region of interest.

$$\begin{bmatrix} X \\ Y \end{bmatrix} \approx \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} q_1 & , & -q_2 \\ q_2 & , & q_1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} , \qquad (1.1)$$

where

p_1, p_2, q_1, q_2 … transformation coefficients.

2.3 Given quantities

2.3.1 Coordinates of ground control points (GCPs)

x_j, y_j … input coordinates of j-th GCP, $j \in J$,

X_j, Y_j … reference coordinates of j-th GCP, $j \in J$,

J … an index set of GCP's identifiers, e.g. $J = \{1, 2, \ldots, n\}$, $n \in \mathbb{N}$.

2.3.2 Accuracy of ground control points (GCPs)

$\sigma_{xy,j}$ … standard deviation of input coordinates of j-th GCP, $j \in J$,

$\sigma_{XY,j}$ … standard deviation of reference coordinates of j-th GCP, $j \in J$,

Normal distribution with rotationally symmetric probability density function is assumed about positions of GCPs in both coordinate systems.

3. PROBLEM SOLUTION

Two principal problems occur while appropriate transformation model is searched for. Firstly, suitable transformation model has to be chosen to express the basic relationship between input and reference coordinates of corresponding points. Such a basic transformation model should be chosen with respect to physical circumstances of capturing the given images, e.g. position of the camera, its inner construction or outer conditions of broadcast of electromagnetic waves. Some simple approximate transformation model is usually applied instead of a rigorous complicated one. Secondly, irregular deformations of the given images can negatively influence suitability of the chosen basic transformation model. Such deformations have to be embodied in the transformation model although they are unknown. They can be treated as a result of some random errors when sufficient number of control points is available. The both problems can be solved simultaneously by means of collocation method.

3.1 Collocation method

Collocation method is well known among geodesists since the early 70's (see [3]), but its origin is much older. The method of collocation originates from the theory of stochastic processes and time series. Similar method was also introduced in 1951 by Dr. Krige and therefore it is called kriging, namely in geostatistics. It is almost equivalent to collocation.

The main principle of collocation is decomposition of the position of a common point in the reference coordinate system into two components: *trend* and *signal*. These two components corresponds to the two above mentioned principal problems. Thus *trend* means the basic transformation model that approximately describes the relationship between input and reference coordinate systems. *Signal* stands for the irregular deformations of the given images. The *signal* actually represents correction of the *trend* to obtain better coincidence of GCPs than the basic transformation model can offer. The *signal* is treated as random process.

The basic transformation model has to be similarity transform due to requirement (1.1). The similarity transform can be concisely formulated with the aid of complex representation of coordinate pairs X, Y, resp. x, y.

$$W = X + \mathbf{i}\,Y\,, \quad w = x + \mathbf{i}\,y\,, \quad W, w \in \mathbb{C}\,,$$

where $\mathbf{i}$ stands for imaginary unit, $\mathbf{i} = \sqrt{-1}$, and $\mathbb{C}$ is set of the all complex numbers. Hence, similarity transform can be expressed as a simple equation:

$$W = p + q\,w\,. \tag{1.2}$$

Variables $p, q \in \mathbb{C}$ are transformation parameters

- p … translation of the both coordinate systems (complex number),
- q … scale and rotation (complex number).

To improve flexibility of equation (1.2), random correction of similarity transform, say $\varphi(w)$, has to be added.

$$W + \varphi(w) = p + q\,w\,, \tag{1.3}$$

where
φ … *signal* - random correction (random complex function).

Equation (1.2) has to be fulfiled for the control points too. Hence

$$W_j + \eta_j + \varphi(w_j) = p + q(w_j + \varepsilon_j)\,, \quad j \in J\,, \tag{1.4}$$

ε_j … measurement error of input coordinates of j-th GCP (complex random variable),
η_j … measurement error of reference coordinates of j-th GCP (complex random variable),
$\varphi(w)$ … signal at a common point,
$\varphi(w_j)$ … signal at the j-th GCP.

Equations (1.3), (1.4) for unknown parameters W, p, q constitute system of equations that has to be adjusted by method of collocation. These equations have to be linearized to separate the unknown parameters from measured quantities.

$$\begin{aligned} \Delta p + \Delta q\; w + \Delta W &= \varphi(w) \\ \Delta p + \Delta q\; w_j + W_j^o - W_j &= \eta_j - q^o\;\varepsilon_j + \varphi(w_j)\,, \quad j \in J; \end{aligned} \tag{1.5}$$

where

$$\begin{aligned} W &= W^o + \Delta W \\ p &= p^o + \Delta p \\ q &= q^o + \Delta q \\ W^o &= p^o + q^o\,w \\ W_j^o &= p^o + q^o\,w_j\,, \quad j \in J\,. \end{aligned}$$

Probability distribution of random vectors $[\varepsilon_1,\ldots,\varepsilon_n]$, $[\eta_1,\ldots,\eta_n]$, $[\varphi(w),\varphi(w_1),\ldots,\varphi(w_n)]$ can be characterized by their covariance matrices $\mathbf{C}_\varepsilon$, $\mathbf{C}_\eta$, $\mathbf{C}_\varphi$. If these covariance matrices are given in advance, unknown parameters ΔW, Δp, Δq can be estimated by ordinary least-squares method. Then, after omitting unknown parameters Δp, Δq, the required coordinates of a transformed point can be expressed as a complex number $\widehat{W}$:

$$\widehat{W} = W^o + \mathbf{c}_w \cdot \mathbf{P} \cdot (\mathbf{W} - \mathbf{W}^o - \mathbf{A} \cdot \widehat{\Delta \mathbf{h}}) + \mathbf{a}_w \cdot (\widehat{\Delta \mathbf{h}} + \mathbf{h}^o)\,, \tag{1.6}$$

where

$\mathbf{c}_w$ … first row of matrix $\mathbf{C}_\varphi$ without the first element of the row,

$\mathbf{P}$ … weight matrix, $\mathbf{P} = \left(\mathbf{C}_\eta + q^o\,\overline{q^o}\,\mathbf{C}_\varepsilon + \mathbf{C}_\varphi^{\square}\right)^{-1}$,

$\mathbf{C}_\varphi^{\square}$ …submatrix of $\mathbf{C}_\varphi$ after omitting first row and first column of $\mathbf{C}_\varphi$,

$\mathbf{W}$ … complex vector, $\mathbf{W} = [W_1, W_2, \ldots, W_n]^T$,

$\mathbf{W}^o$ …complex vector of approximate coordinates, $\mathbf{W}^o = [W_1^o, W_2^o, \ldots, W_n^o]^T$,

$\mathbf{A}$ … design matrix, $\mathbf{A} = [\mathbf{1}_n, \mathbf{w}]$,

$\mathbf{1}_n = [\underbrace{1, 1, \ldots, 1}_{n}]^T$,

$\mathbf{w}$ … complex vector, $\mathbf{w} = [w_1, w_2, \ldots, w_n]^T$,

$\widehat{\Delta\mathbf{h}} = (\mathbf{A}^\sharp \mathbf{P} \mathbf{A})^{-1} \mathbf{A}^\sharp \mathbf{P} (\mathbf{W} - \mathbf{W}^o)$,

$\mathbf{A}^\sharp$ … complex conjugate of $\mathbf{A}^T$, $\mathbf{A}^\sharp = \overline{\mathbf{A}^T}$,

$\mathbf{a}_w$ … similarity transform operator, $\mathbf{a}_w = [1, w]$,

$\mathbf{h}^o$ … approximate coefficients of similarity transform, $\mathbf{h}^o = [p^o, q^o]^T$.

Real components $\widehat{X}$, $\widehat{Y}$ of complex number $\widehat{W}$ computed by (1.6) are the required reference coordinates of a transformed point. The resulting transformation model is as follows.

$$\mathbf{t}(x, y) = [\widehat{X}, \widehat{Y}]. \tag{1.7}$$

Transformation $\mathbf{t}$ is conform because formula (1.6) defines complex function of complex argument. Such a function (so called holomorphic function) has been proved to represent conformal mapping (see [4], theorem 8.2).

3.2 Image registration

Collocation method described in the previous section can be easily applied to registration of digital images. Transformation formula (1.6) can be evaluated for each pixel of the input image. This straightforward application brings problem with assignment of colors to pixels of the transformed image, since the transformed pixels create irregular grid. Therefore proper assignment of colors needs additional interpolation in the irregular grid, especially in case of significant nonlinear deformation of images. To avoid the interpolation, method of nearest neighbor can be applied instead. This method assigns to a pixel $[X, Y]$ of the input image the color of pixel $\mathbf{t}^{-1}(X, Y)$ from the input image. It means that inverse mapping $\mathbf{t}^{-1}$ has to be computed to transform the input image. Inversion of complicated formula (1.6) need not be computed since much simpler way exists to obtain $\mathbf{t}^{-1}$. It is more suitable to simply exchange input and reference coordinate systems and apply collocation method by the same manner as in the forward case.

3.3 Precision of the registration

Positional precision of the registration is characterized by standard deviation $\sigma_{XY}(X,Y)$ which can be computed for any point $[X,Y]$ in the reference image.

$$\sigma_{XY}(X,Y) = \sqrt{\sigma^2 - \mathbf{c}_w \mathbf{P}\mathbf{c}_w^T + (\mathbf{a}_w - \mathbf{c}_w \mathbf{P}\,\mathbf{A})(\mathbf{A}^\sharp \mathbf{P}\mathbf{A})^{-1}(\mathbf{a}_w - \mathbf{c}_w \mathbf{P}\mathbf{A})^\sharp}\ . \tag{1.8}$$

Standard deviation $\sigma_{XY}(X,Y)$ depends on two statistical parameters. These parameters control fitting degree of GCPs. The both parameters influence covariance matrix $\mathbf{C}_\varphi$ through covariance function. One of the parameters, σ, characterizes probability distribution of difference between transformation $\mathbf{t}$ and similarity transformation. This probability distribution is assumed to be normal with variance σ^2. Optimal values of the statistical parameters can be optionally entered by the user or estimated by Bayesian approach.

To evaluate formula (1.8) for some given point $[X,Y]$ in the reference image, corresponding input coordinates $[x,y]$ have to be computed first.

$$[x,y] = \mathbf{t}^{-1}(X,Y)\,,$$
$$w = x + \mathbf{i}\,y\,.$$

Computation of inverse transformation $\mathbf{t}^{-1}$ is described in section 3.2. After the inverse transformation and after setting up an optimal value of parameter σ, formula (1.8) can be evaluated.

3.4 Software implementation

The proposed method of image registration was implemented as a web application by means of up-to-date software standards of Internet technology. The main procedure which evaluates formulae (1.7) and (1.8) is written in C++. Special library for complex arithmetics was used to code formulae (1.7) and (1.8) easily. Other server-side modules were programmed in Python language with the aid of web framework Django. Client-side software is based on HTML and SVG standards, JavaScript support is utilized as well.

The user can load his own images into the web application or use Web Map Services (WMS). Precision of the registration can be shown globally by isolines of function σ_{XY} or locally by a circle of radius $\sigma_{XY}(X,Y)$ at point where the user has clicked by his mouse.

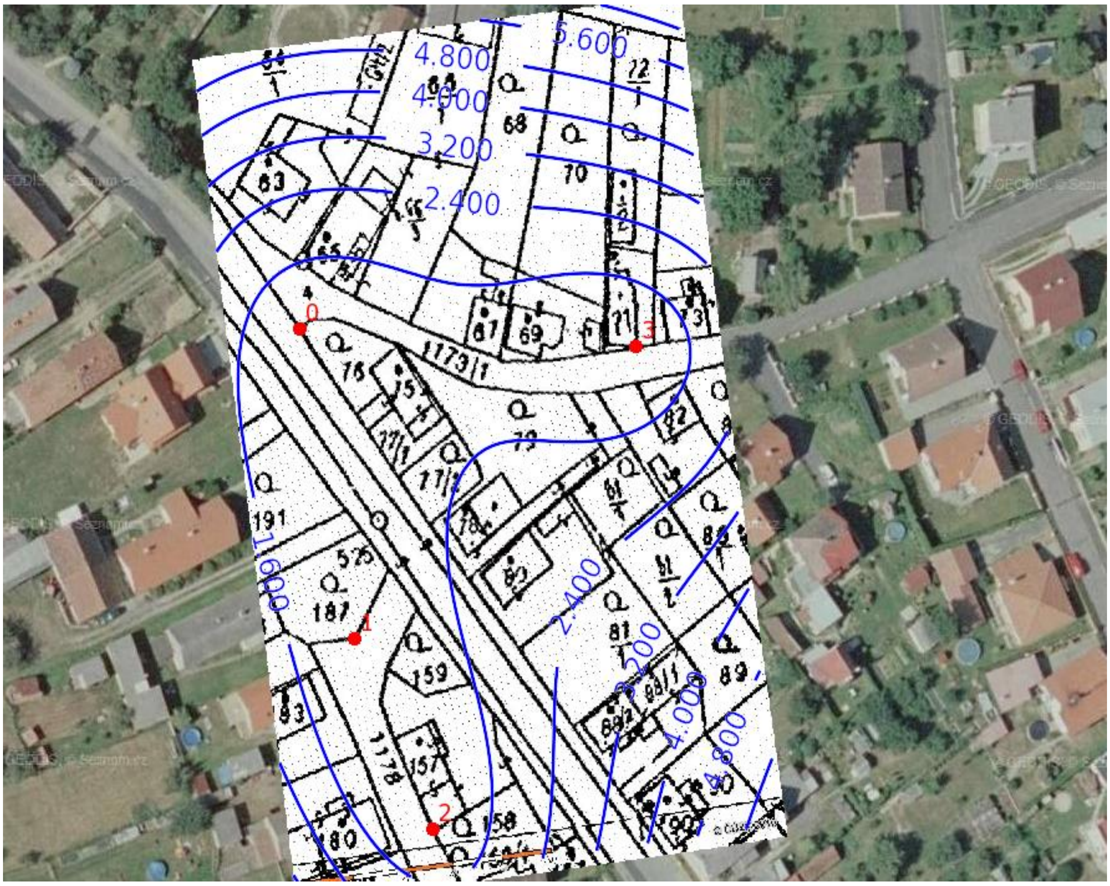

Figure 1: Registration of cadastral map into orthophoto

Typical use of the web application is shown on Figure 1. The white rectangle is part of cadastral map that is registered into orthophoto. GCPs are marked by red points, the blue curves are isolines of same positional accuracy.

The application is freely available at http://www.vugtk.cz/igc/apps/transformation/ for any Internet user. The user has to register at http://www.vugtk.cz/~deformace/pgm/registration/index.php.

4. CONCLUSION

The proposed registration method has several advanced features that make it unique among other existing methods, namely:

1. Positional precision of any transformed pixel in the registered image can be estimated without need of ground truth.
2. Transformation between images is conformal (preserves angles).

3. All the tunable parameters of the transformation model have real interpretation: geometrical or statistical.
4. Positional biases at GCPs are optimally spread out in the area of interest to avoid overfitting. (Immoderate distortion of input image caused by forced fit of GCPs is restrained.)
5. Smoothness of the transformation is robust to configuration of GCPs. (Non-uniform distribution of GCPs in the area of interest does not matter.)

The proposed method of image registration was implemented as a web application which is freely available at http://www.vugtk.cz/igc/apps/transformation/ for any Internet user.

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BIOGRAPHICAL NOTES

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