

# The Estates Valuation Models in the Developing Markets\*

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**Key words:** real estates' valuation, statistical methods, conditional estimation model

## SUMMARY

The authenticated deeds include the transaction prices for the groups of estates or estates with complicated structure. Prices obtained from authenticated deeds neither can be used for market analysis, nor for similar estates value estimation. The need for the whole estate price separation into unit transaction prices appears then.

Finding solution of this problem enables using information from authenticated deeds in the estates market analysis. It also helps in applying this data into similar estates cadastral and market value estimation.

The model of estate transaction price (from authenticated deed) dividing into particular elements prices is presented in this paper. Number of transactions is often smaller than deliberated estate elements number, so the conditional estates unit prices model should be used for estimation.

The deliberated real estates are grouped according to its destination in the local spatial development plan, building type and the building usage. The conditional equations should be set to separate the whole transaction price into elements prices, for every group of similar estates.

The unit prices conditional model gives proper market solutions if unit prices approximations ( $\tilde{c}_i$ ) adequate for the analyzed market are predetermined. Markets differing on localization may be characterized by various real estates unit element prices. The unit prices proportion ratios may be the reference for various estates markets.

In the Polish conditions, determining and collecting information on different estates elements unit prices may be helpful in financial (fiscal) system reform. It can also make changing tax system based on area into one based on value easier. The place for collecting all data necessary for that is the prices and values register for real estates – the integral part of ground and building cadastre.

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## 1. INTRODUCTION

Nowadays, authenticated deeds contain transaction prices for real estates sets or for urbanized estates with complicated structure. Such deeds can be used neither for market analysis, nor for similar estates market value estimation. For such authenticated deeds, there is a need to divide the whole estate value into unit transaction prices of all estates elements.

For estates that are rarely subjects of trade, the model of transaction prices from authenticated deeds separation into unit prices is presented in this paper. The conditional model of real estates parts unit prices should be used in the value estimation process.

To divide transaction price into separate estates parts prices, for each group of similar estates, it means having the same destination in local spatial plan, type or function of buildings, erected on the ground, it is necessary to set equations for each estates group. The conditional model of unit prices is going to give right market solutions, when the proper approximated unit prices for analysed market ( $\tilde{c}_i$ ) estates elements will be presumed. The real estates markets differing on their localization can have the different values of unit prices for real estates elements. The reference for different real estates markets could be coefficients of mutual proportion for estates elements unit prices.

## 2. THE MODEL CONSTRUCTION

The transaction number is often smaller than number of elaborated elements for estates being rarely subject of trade, so applying conditional model of unit prices for estates elements is necessary for the estimation process. To divide transaction price into separate estates parts prices on criteria of their destination in local spatial plan and type or function of buildings, it is necessary to set equations for each estates group. Generally, the equation has the following formula

$$S_1 \cdot (\tilde{c}_1 + \delta_1) + S_2 \cdot (\tilde{c}_2 + \delta_2) + \dots + S_i \cdot (\tilde{c}_i + \delta_i) = C_T \quad (1)$$

These symbols are explained below:

$C_T$  – the transaction price of elaborated estate,

$S_i$  – the area of every i-element (parcel, parcel parts having defined soil classes, flat or building usable areas or whole building),

$\tilde{c}_i$  – i-element approximated unit price,

$\delta_i$  – i-element approximated unit price random remainder.

If we multiply and group the similar objects in formula (1), we will receive the conditional model for estates unit prices as follows

$$S_1 \cdot \delta_1 + S_2 \cdot \delta_2 + \dots + S_i \cdot \delta_i = C_T - S_1 \cdot \tilde{c}_1 - S_2 \cdot \tilde{c}_2 + \dots - S_i \cdot \tilde{c}_i. \quad (2)$$

This equation constant term is the difference between transaction price ( $C_T$ ) and model price ( $C_M$ ) of deliberated real estate. The model estates price is calculated from areas multiplication products and approximated unit prices particular estate values. So the formula has the following form

$$C_M = S_1 \cdot \tilde{c}_1 + S_2 \cdot \tilde{c}_2 + \dots + S_i \cdot \tilde{c}_i. \quad (3)$$

The estates prices conditional model application is going to give proper results only if the appropriate estates components prices approximations ( $\tilde{c}_i$ ) have been calculated earlier. The estates components prices vary for different real estates markets. Coefficients of mutual proportion for estate elements unit prices can make the reference level for differing estates markets.

The proportion coefficients of estates elements unit prices structure can be delivered from replacement value analysis. In order to achieve it, the average replacement values for unit of estates element and then coefficients of mutual proportion should be elaborated. All this ought to be done not only for each building type, but land as well. The coefficients  $u_i = u(i)$  of element prices shares in the whole estate price should be delivered earlier. When elaborating these values some specifics of local markets should be taken into account.

The approximated values ( $\tilde{c}_i$ ) of urban estates elements unit prices can be elaborated using the following procedure:

- the value of coefficient ( $u_i$ ) should be found for each element of elaborated estate (land and other elements) and the replacement value should be taken into account,
- the multiplication results of area ( $S_i$ ) and coefficient ( $u_i$ ) should be calculated for each element of elaborated estate,
- the sum of multiplication results ( $S_i \cdot u_i$ ) should be calculated for every elaborated estate,
- the coefficient ( $U_j$ ) of prices range should be calculated for every elaborated estate. It is the result from dividing the transaction price by the sum of multiplication results. The formula stands as follows

$$U_j = \frac{C_{Tj}}{\sum_i S_{ji} \cdot u_{ji}}, \quad (4)$$

- the range of variation coefficients ( $U_j$ ) should not exceed 20 % of its average value. If this condition is not satisfied, the values of coefficients ( $u_i$ ) should be corrected in their ranges or the exceptional estates values should be omitted,
- the delivered minimal value of coefficient ( $U_j \Rightarrow (U_{\min} = U_M)$ ) will make the coefficient of model prices range for elaborated estates group,
- the approximated values ( $\tilde{c}_i$ ) unit prices of estates elements makes the multiplication results of coefficients ( $u_i$ ) elaborated for estates elements and coefficient ( $U_M$ ) of model prices range

$$\tilde{c}_i = u_i \cdot U_M . \quad (5)$$

Applying calculated values (5) we may find model constant term (2), for which we obtain the following formula

$$\Delta C_T = C_T - S_1 \cdot \tilde{c}_1 - S_2 \cdot \tilde{c}_2 + \dots - S_i \cdot \tilde{c}_i = C_T - C_M . \quad (6)$$

When taking (6) into account, the conditional model of elements unit prices for elaborated estates having known transaction price will stands as follows

$$S_1 \cdot \delta_1 + S_2 \cdot \delta_2 + \dots + S_i \cdot \delta_i = \Delta C_T . \quad (7)$$

The set of equations (7), formulated as a matrix for several transaction prices will have the formula

$$\begin{bmatrix} S_{11} & S_{12} & \dots & S_{1i} \\ S_{21} & S_{22} & \dots & S_{2i} \\ \vdots & \vdots & \dots & \vdots \\ S_{j1} & S_{j2} & \dots & S_{ji} \end{bmatrix} \times \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_i \end{bmatrix} = \begin{bmatrix} \Delta C_{T1} \\ \Delta C_{T2} \\ \vdots \\ \Delta C_{Tj} \end{bmatrix} , \quad (8)$$

where

- indices 1÷i point out elements of similar estates elaborated group,
- indices 1÷j point out the following number of similar estates sell transaction number.

The conditional model that is shown as a set of equation (8), have the solution when the transaction number (j) is smaller than number of elaborated elements (i).

The random remainders ( $\delta_i$ ) of approximated values ( $\tilde{c}_i$ ) for estates unit prices are the estimated values of this model.

The set of equations (8) have the following form in the matrix notation

$$\{S_w\} \cdot \{\delta\} = \{\Delta C\}, \quad (9)$$

where

$\{S_w\}$  – orthogonal horizontal matrix that consists of elaborated estates area,

$\{\delta\}$  – random remainders of approximated values for estates elements unit prices,

$\{\Delta C\}$  – matrix made of results of subtraction between transaction and approximated model prices for the whole estates.

The set of equations meets assumptions of Gauss-Markov model, so

$$E[\{S_w\} \cdot \{\delta\}] = \{0\} \Rightarrow E[\delta] = 0. \quad (10)$$

The estimators of random remainder deviation vector  $\{\hat{\delta}\}$  and vector  $\{\hat{c}_i\}$  of estates elements unit prices and remainder variance estimator ( $\hat{\sigma}_0^2$ ), the covariance matrix of vector  $\{\hat{\delta}\}$  and the covariance matrix of vector  $\{\hat{c}_i\}$ , may be calculated from the following formulas:

$$\{\hat{\delta}\} = \{S_w\}^T \left( \{S_w\} \cdot \{S_w\}^T \right)^{-1} \cdot \{\Delta C\} \quad (11)$$

$$\hat{c}_i = \tilde{c}_i + \delta_i \quad (12)$$

$$\hat{\sigma}_0^2 = \frac{\{\hat{\delta}\}^T \{\hat{\delta}\}}{j} \quad (13)$$

$$Cov\{\hat{\delta}\} = \hat{\sigma}_0^2 \left( \{S_w\}^T \left( \{S_w\} \{S_w\}^T \right)^{-1} \{S_w\} \right) \quad (14)$$

$$Cov\{\hat{c}\} = \hat{\sigma}_0^2 \left( \{I\} - \{S_w\}^T \left( \{S_w\} \{S_w\}^T \right)^{-1} \{S_w\} \right). \quad (15)$$

The estimated unit prices of estates elements meet the assumptions of conditional models for the transaction prices of whole deliberated estates.

### 3. RECAPITULATION

The conditional model presented in the paper enables the separation of the whole transaction price into the prices of elementary estate parts. Its application for any types of estates requires prior calculation of unit prices of estates elements mutual proportion coefficients. These coefficients will be used for the proper calculation of approximate values for estates elements unit prices that are required in the conditional model.

The calculation of unit price for every estates component enables us using information from authenticated deeds in the real estates market analysis. Moreover, it provides us with information for estimation of similar estates market and cadastral values.

Calculating, processing and managing information concerning unit prices of estates elements may be important step in fiscal system reform in Poland. It can also be very supportive in tax changes from tax based on area into one based on value.

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## BIOGRAPHICAL NOTES

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