Geometry of "The Struve Arc" Compared With Up–to–date Geodetic Data

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Key words: meridian arc, principal points, coordinates

SUMMARY

The paper presents main outcomes of the second comparison between up–to–date geodetic data and the final results of the measurements of the Russo–Scandinavian meridian arc of 25° 20′ carried out during 1816–1855. To make this mathematical comparison was the goal of an extensive investigation performed under the aegis of *The St.Petersburg Society for Surveying & Mapping*, within the frames of an action of commemoration of the 150th anniversary of publication of *"Arc du Méridien de 25°20′ entre le Danube et la Mer Glaciale"* by F.G.W. Struve in 1857. The required input data have been submitted by member agencies of the international Coordinating Committee managing the World Heritage "The Struve Geodetic Arc", as well as taken from many additional sources, incl. author's previous research works. The investigation embraced items of the methodology, thorough examination of historical identity of the 13 principal meridian arc points, computations and interpretation.

The main result obtained is differences in length and azimuth between eight Struve's (of his total 12) geodesics and the adequate modern ones linking the principal arc points. These are, for the subsequent Struve Nos V (*Belin – Nemesch*) to XII (*Stuoroivi – Fuglenaes*): – 7.3 m/ + 5.5 as; – 5.7 m/ + 2.4 as; + 2.3 m/ + 1.6 as; – 1.7 m/ + 1.7 as; – 0.0 m/ + 0.1 as; – 0.4 m/ + 0.9 as; – 9.9 m/ + 8.0 as; – 11.4 m/ + 12.9 as; the values mean "1857 minus 2007", "as" is "arcseconds". Not simple relations of these values to *residual errors* of the respective measuring technologies of the historic 40–year work have been determined. It is worthy of note that the first general examination of the "geometry" of "the Struve arc" made in 1994 by means of GPS–measurements exactly accords with the new results obtained for the four segments of the arc northern branch from *Mäki–päälys* to *Fuglenaes*.

Historical background of "The Struve Arc" measurements is explicated, and a far going scientific implication of Struve's treatise demonstrated by a great deal of sources. The results obtained will hopefully promote better comprehension of the *geodetic contents* of the World Heritage *"The Struve Geodetic Arc"*, as well as preservation of its 13 principal points.

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PREFACE

In August 2006 *The St.Petersburg Society for Surveying & Mapping* board addressed to the international Coordinating Committee managing the World Heritage *"The Struve Geodetic Arc"* (SGA) with an initiative to undertake a joint action in connection with the 150–year anniversary of the first publication by academician F.G.W.Struve of his eminent work *"Arc du Méridien de 25°20' entre le Danube et la Mer Glaciale"* in 1857. Comparative research into Struve's main geodetic results and a subsequent commemorative publication [Kaptüg, 2007] is the outcome of this action; the goal was reached thanks to Committee's approval and cooperation of its member geodetic agencies of Estonia, Belarus, Norway, Finland, Lithuania, Latvia and Sweden.

Input data on the 13 principal points of the SGA were required to fulfill the mathematical part of the undertaken work. These were submitted with except for only the Struve arc southern segments, in the course of 2007; a considerable amount of additional material was also used: archive documents found in Russia, Sweden and Norway, results of exploratory and recovery works carried out in Russia and Belarus, a lot of publications. There followed studies in methodology of the scheduled comparison, examination of identity of the submitted foreign data, tracing the history of use of the 13 principal SGA points, computations and interpretation of the results obtained.

1. HISTORICAL BACKGROUND

By the middle of the 18th century direct geodetic measurements had definitively confirmed the reduction of the meridian degree length at the Earth's Equator. The *terrestrial globe* gave way to the *terrestrial spheroid* and the shape of the latter became a permanent theme for global science, and the object of studies by mathematicians, astronomers and surveyors, and the aim of numerous *degree measurements* at Earth's different meridians and parallels. During these works more and more advanced methods and instruments to measure angles and base lines, and to determine latitudes and azimuths were used; as a result, the parameters of the Earth shape became clearer with gradually increasing accuracy. Ground meridian arc measurements went on almost until the middle of the 20th century.

What was the need of all these newer and newer measurements against more or less established and continuously refined values of the Earth shape parameters?

– First of all they were of scientific importance. Further refining of values of the Earth semi–axis and flattening contributed to the development of the interrelated system of astronomical constants; without this knowledge it would have been impossible to combine the phenomena observed in the Solar system and Universe and build an integral physical pattern of the world. Then, the continuous work of surveyors offered new data to improve knowledge of the geometry, physics and dynamics of the Earth material. For this purpose various measurements were carried out in different areas on the globe and they are still obtained today where modern technologies permit the continuous monitoring of geodynamical changes of the Earth surface in the range of accuracy from decimetres to millimetres.

– Then, it had a practical utility for national mapping, especially when it was required to *transfer coordinates* which were to become the mathematical basis of military topographic maps and navigational charts. Observed triangular frames of meridian arc measurements (chains of adjacent triangles) provided even extra accuracy for such a purpose. The prospect of getting coordinate positions aroused a keen interest at army and navy headquarters in scientific measurements. "The Struve arc" was not an exception. Army officers of Russia, Sweden and Norway have conducted a major part of the angular measurements in its triangles all the way from the Arctic Sea to the delta of the Danube river.

– Lastly, they had an international public importance similar to that of space research in the second half of the 20th century. Degree measurements used to be "*great science*" [Batten 1988] of the 18th and 19th centuries being a topic of active international contacts between scientists, the military and statesmen, even including monarchs. Participation in arc measurements certified the resources of the States involved and enhanced their political and military prestige.

The Russo–Scandinavian meridian arc measurement was a major contribution to investigations of the Earth shape that had been made by the end of the 19th century. From an historical viewpoint it was in fact a first measurement of the shape of our planet taken in Russia. Commencement of the work goes back to 1816 when the Russian Colonel Carl F. Tenner in response to his initiative was allowed (without any additional costs !) to combine the forthcoming triangulation of the Vilna province with measure of the meridian arc passing through the Vilna observatory (now in Vilnius, Lithuania). In 1819 Alexander I agreed to finance another (this time purely scientific) project of the meridian arc measurements, that was advanced for the area of Livland and Estland by F.G.W. Struve who was at the time an "extraordinary" (out–of–staff) professor of astronomy of the Dorpat university (now in Tartu, Estonia). Struve and Tenner in spite of their numerous duties and various obstacles took up and accomplished a toil of organization, fulfillment and calculation of field geodetic and astronomical operations and publication of corresponding descriptions. Their measurements combined in a single *"Russian arc"* having the southern terminal at the Danube delta and the northern one in Tornea at the Gulf of Bothnia; at the final stage the operations embraced also northern Scandinavia where they were prolonged up to the Arctic Sea, based on collaboration with the Swedes and Norwegians. All in all works lasted for 40 years: final field measurements were carried out in August 1855. Computation was totally completed at the end of 1856 when Struve handed over his manuscript of the Volume II of *«Arc du méridien de 25° 20′...»* to the publishers. Finally the volumes I and II describing the geodetic part of the 40–year works with relating drawings (*"Planches"*) were published in the St. Petersburg academy printing house [Struve 1856–1857]; Struve soon presented his treatise to the Paris Academy of Science (on October 12, 1857, [Struve 1857]). Most probably sensing weakness of his health and being uncertain about publication of the closing volume (due to embrace a detailed account of the astronomical part of the works and final deduction of the Earth shape parameters) Struve handed over his treatise to Lt.–Colonel H. James, *Superintendent* of the UK *Ordnance Surveys*. Under James's command Captain A.R. Clarke at that time was deriving new parameters of the terrestrial ellipsoid based on the world's most reliable meridian arc measurements. Clarke's conclusion using data of the complete 25–degree "*Russian arc"* was published in the following year [Clarke 1858].

2. SCIENTIFIC IMPLICATIONS OF THE MEASUREMENTS

Thanks to a constant initiative, patriotic aspirations and tireless work of Struve, Tenner and their colleagues in Russia, Sweden and Norway this outstanding scientific and technical achievement became real and held a universal priority. In specialist literature the scientific virtues of "the Struve arc" were appreciated in detail more than once:

– a great 3000–km extension of the measurements that leveled the influence of local disturbances of the vertical line along the triangle chain,
– continuous analysis of accuracy until the probable error of each resulting value is deduced – "*an advantage which no other arc possesses*" [Clarke 1858],
– performance of the measurements far beyond the Arctic Circle,
– numerous scientific findings and techniques that became hereditary tools of the *world astronomic–geodetic school*.

The most advantageous geographic location, great length of the arc and its high accuracy were the main features that contributed to a continuous scientific value of "the Struve arc" and ensured its long scientific "life". Accurate astronomical and geodetic data of continental extent became an indispensable material for further scientific studies of the mathematical shape of the Earth.

More expressively than many words the scientific implications of "the Struve arc" can be demonstrated by the following list [Kaptüg, 2007]. It presents, in a chronological order the authors of the most important studies relative to the mathematical shape of the Earth that were performed during 1810–1960 and based on available astronomical–geodetic data, including *astronomical–geodetic arcs*. Here one can see that from 1829 on "*the Russian*" (later *"the Russo–Scandinavian"*) meridian arc had been continuously requested for *almost all* the studies of the mathematical shape of the Earth by the method of comparison of various astronomical–geodetic arcs, – until this very method left the stage by giving way to satellite–based technologies.

Authors	when completed	REFERENCES TO USING DATA FROM *"THE STRUVE GEODETIC ARC"*
Delambre	1810	–
Bowditch	1817	–
Walbeck	1819	–
Airy	1826	–
Schmidt	1829	–

In March 1829 the main results of the first completed segment of 3°.6 of the future *"Russian Arc"* are published by Struve in *Astronomische Nachrichten*, 1829, *7, 164*: 385–400; a two-volume account is out of print in early 1832 (*"Beschreibung…"*, dated *"1831"*).		
Airy	1830	"Figure of the Earth" – *Encyclopaedia Metropolitana…,* London,1835, *Mixed Sciences*, *III*: 218–220 *[reprinted in the further editions]*.
Everest	1830	– *this research was based on the Indian surveys only*.
Schmidt	1831	"Von den Dimensionen der Erde" – *Astronomische Nachrichten*, 1831, *9, 209*: 315–316, *also in* *Astronomische Nachrichten,* 1831, *9, 213:* 371–372.
By the end of 1832 the Struve account on a linkage of both meridian arcs of the "*Russian measurements of degrees"* is out of print in St. Petersburg (in German); in January 1833 – also in *Astronomische Nachrichten*,1833, *10, 236*: 323–332.		
Bessel	1834	Bessel's mathematical investigation, within his letter to Gen. Tenner; *a copy in the manuscript titled* *"Beschreibung der Breitengradmessungen …,* von *C.F.Tenner, 1834*", vol. *II*: 559–578; *a Russian version in* *Записки Военно–топографического Депо*, Санктпетербург,1844, *IX*: 905–922.
Bessel	1837 1841	"Bestimmung der Axen des elliptischen Rotationssphäroids…" – *Astronomische Nachrichten*, 1837, *14, 333*: 334–337; *corrected in* *Astronomische Nachrichten*, 1842, *19, 438*: 115.
Everest	1847	*Derivation of the Earth's shape parameters* – An account of the measurements of two sections of the meridional arc of India…, London, 1847: 425–431.
Paucker	1853	"Die Gestalt der Erde" – *Bulletin de la Classe phys.–math. de l'Acad. Imp. de Sciences de St.Pétersbourg*, 1853, *12*: 635–636.
By Mai 1853 Struve obtains the results covering the entire *"Russian Arc"* of 20°.5 from Izmail to Tornea.		
Struve	1854	*"Untersuchungen über die Dimensionen der Erde"* – *a manuscript of May 1853 in* The St.Petersburg Branch of the Archives of the Russian Academy of sciences, code *2.1/1850.2:* 66–69; *the results are presented in* Arc du méridien…, tome I, 1856 (*also* 1860, 1861): 82–84.
Clarke	1856	*Derivation of the Earth's shape parameters* – "On the figure, dimensions…of the Earth", by H. James – *Philosophical Transactions,* R. Society of London, 1856, *146*: 620, 623–624.
In February 1856 (vol. I) and in September 1857 (vol. II and diagrams) Struve's treatise *"Arc du méridien de 25°20′…"* is out of print; the 2nd (added) edition in French is published by the end of 1860.		
Clarke	1858 1863	*Derivation of the Earth's shape parameters* – Account of the observations and calculations of the Principal Triangulation, London, 1858: 752–753, 760–778; *in 5 years the results were corrected after the added 2nd edition of "Arc du méridien…"* – Extension of the triangulation of the Ordnance Survey into France and Belgium, by H.James, London, 1863: p. III.
Schubert	1859	"Essai d'une détermination de la véritable figure de la Terre" – *Mémoires de l'Acad. Imp. de Sc. de St.Pétersbourg*, 1859, VII ser., *I, 6*: 2, 4, etc.
Clarke	1860	"On the figure of the Earth" – *Memoires of the R. Astronomical Society,* London, 1861, *XXIX*: 32–40.
Schubert	1860	"Sur l'influence des attractions locales…" – *Astronomische Nachrichten,* 1860, *52, 1245–1247*: 333–362.
Schubert	1861	"Ueber die Figur der Erde" – *Astronomische Nachrichten*, 1861, *55, 1303*: 103–110.
Pratt	1863	"On the degree of uncertainty which local attraction…occasions…in the mean figure of the earth" – *Proceedings R. Soc. of London*,1864, *XIII*: 266–271, 274.
Villarceau	1866	– *this research was based on the French surveys only.*
Clarke	1866	*Derivation of the Earth's shape parameters* – Comparison of the standards of length…, London, 1866: 283–286.
Ph. Fischer	1868	Untersuchungen über die Gestalt der Erde, Darmstadt, 1868: 125–314.
Pratt	1871	A treatise on attractions… and the figure of the Earth, 4th (the last, added) ed., London and New York, 1871: 169–177.
Clarke	1878	"On the figure of the Earth" – *L.,E.,D. Philosophical Magazine and Journal of*

		Science, series 5, 6, 35, August 1878: 84–88.
Clarke	1880	"Figure of the Earth" – Geodesy, Oxford, 1880: 316–319.
Бонсдорф	1888	«Определение размеров Земли…» – *Записки Военно–топографического Отдела*, Санкт–Петербург, 1888, XLII, II, раздел ix: 6 стр.
Жданов	1892	«Вычисление размеров земного сфероида…» – *Записки Военно–топографического Отдела*, Санкт–Петербург, 1893, L, II : 321–322.
Красовский	1901	«Определение размеров земного трехосного эллипсоида…» – *Памятная книжка Константиновского Межевого института за 1900–1901,* Москва, 1902: 35–54.
Schumann , Helmert	1906	"Die Grösse der Erde" – *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften,* Berlin, 1906, I: 526–527; *also in Verhandlungen der XV Konferenz der Internationalen Erdmessung*, Berlin, 1908: 134, 264.
Hayford	1909	– *this research was based on the USA surveys only.*
Helmert	1913	"Geoid und Erdellipsoid" – *Zeitschrift der Gesellschaft für Erdkunde zu Berlin*, 1913, 1: 28–29.
Heiskanen	1926	"Die Erddimensionen nach den europäischen Gradmessungen" – *Veröffentlichungen des Finnischen Geodätischen Institutes*, Helsinki, 1926, 6: 15–17, 22–26.
Heiskanen	1929	"Über die Elliptizität des Erdäquators" – *Veröffentlichungen des Finnischen Geodätischen Institutes*, Helsinki, 1929, 12: 14–18.
Красовский (Krassowski)	1936	«Обзор и результаты современных градусных измерений» – *Геодезист*, Москва, 1936, 10:12–17, 11: 35–36, 12: 11,15,18–23 ; *also in* Избр. соч., I, Москва, 1953: 126–178; *the first version of 1935 in Verhandlungen der 8ten Tagung der Baltischen Geod. Kommission,* Helsinki, 1936: 179–194.
Изотов	1940	«Форма и размеры Земли по современным данным» – *Труды ЦНИИГАиК*, Москва, 1950, 73: 120–131 – *derivation of the TSNIIGAIK global ellipsoid, afterwards the reference figure was renamed to " the KRASOVSKY ellipsoid ".*
Jeffreys	1948	"The figures of the Earth and Moon" – *Monthly Notices of the R. Astronomical Society, Geophysical Supplement, 1948, 5, 7: 222, 229–232.*
Ledersteger	1949	– *these research works used only the recent material obtained from triangulations of the"Baltic Ring" states which partly overlapped the relatively poorer SGA data.*
Ledersteger	1951	–
Жонголович	1955	«Об определении размеров общего земного эллипсоида» – *Труды Института теоретической астрономии,* Москва–Ленинград, 1956, VI: 41–44.
Chovitz , I. Fischer	1956	– *this research was based only on the data obtained from the four most recent trans–continental astronomical–geodetic arcs.*
Bomford	1956	– *these authors used only the most recent European astronomical–geodetic material available, not including that of the USSR because the latter had not been published.*
Hough	1956	–
Oxford	1959	–
I. Fischer	1960	–

3. MATHEMATICAL SHAPE OF THE EARTH

In 1853–1854 Struve computed preliminary values of probable dimensions of the mathematical shape of the Earth using the world's two longest meridian arc measurements – Russian and English (in India). Having combined the acquired solution with then the most justified deduction of Bessel (1841) Struve derived new parameters of the biaxial Earth ellipsoid which have been saved in the chronicles of Geodesy under the name "*the Struve ellipsoid*". These values, within the limits of their probable errors deduced by Struve stand out against all the previously deduced values because at that time they provided the best fit to the global biaxial ellipsoid of revolution whose precise parameters are well–known today:

Struve's 1853–1854 parameters of the biaxial global ellipsoid [Struve 1856, p. 84]:
mean length of 1° of a meridian arc, in toises: 57019.75 ± 1.15 prob. err.;
flattening denominator: 294.73 ± 1.72 prob. err.;

Modern parameters of the biaxial global ellipsoid:
mean length of 1° of a meridian arc: 111132.95 m, in toises: 57018.6;
flattening denominator: 298.25722.

4. GEODETIC DATA TO BE COMPARED

The following table 1 presents Struve's summary of the final geodetic results of the Russo–Scandinavian meridian arc measurements. It contains length values of the 12 closing geodetic lines ("geodesics"), their probable errors, latitude and spheroid azimuth values taken at the end points of 12 segments of the triangulation chain built up along the Dorpat observatory meridian over an extent of 3000 km.

Table 1. Final geodetic results of the Russo–Scandinavian measurements [Struve 1856–1857].

SGA segments	End points of the SGA segments	Preliminary latitudes	Closing geodesics: lengths and their probable errors, in toises	Spheroid azimuths
I	***Staro–nekrassowka*** ***Wodolui***	φ = 45° 20′ 02″.8 φ = 47° 01′ 25″.2	**r** = 96580.94 ± 0.650	**U** = 3° 18′ 24″.988 **B** = 183° 24′ 35″.737
II	***Wodolui*** ***Ssuprunkowzi***	φ = 47° 01′ 25″.2 φ = 48° 45′ 03″.1	**r** = 131597.24 ± 0.788	**U** = 319° 19′ 49″.834 **B** = 137° 38′ 40″.272
III	***Ssuprunkowzi*** ***Kremenetz***	φ = 48° 45′ 03″.1 φ = 50° 05′ 50″.0	**r** = 87009.17 ± 0.649	**U** = 332° 18′ 43″.009 **B** = 151° 28′ 28″.714
IV	***Kremenetz*** ***Belin***	φ = 50° 05′ 50″.0 φ = 52° 02′ 42″.2	**r** = 112544.59 ± 1.001	**U** = 351° 22′ 51″.580 **B** = 171° 00′ 30″.293
V	***Belin*** ***Nemesch***	φ = 52° 02′ 42″.2 φ = 54° 39′ 05″.9	**r** = 148848.20 ± 1.425	**U** = 1° 16′ 01″.624 **B** = 181° 20′ 48″.708

VI	***Nemesch*** ***Jacobstadt***	φ = 54° 39′ 05″.9 φ = 56° 30′ 04″.8	**r** = 107163.31 ± 0.714	**U** = 9° 09′ 34″.103 **B** = 189° 36′ 16″.160
VII	***Jacobstadt*** ***Dorpat***	φ = 56° 30′ 04″.8 φ = 58° 22′ 47″.6	**r** = 110528.46 ± 0.642	**U** = 13° 33′ 57″.606 **B** = 194° 17′ 39″.622
VIII	***Dorpat*** ***Mäki–päälys***	φ = 58° 22′ 47″.6 φ = 60° 04′ 29″.4	**r** = 97168.28 ± 0.499	**U** = 4° 12′ 37″.313 **B** = 184° 25′ 29″.636
IX	***Mäki–päälys*** ***Kilpi–mäki***	φ = 60° 04′ 29″.4 φ = 62° 38′ 05″.0	**r** = 146464.23 ± 1.07	**U** = 357° 55′ 09″.337 **B** = 177° 44′ 31″.144
X	***Kilpi–mäki*** ***Tornea***	φ = 62° 38′ 05″.0 φ = 65° 49′ 44″.7	**r** = 193965.33 ± 1.33	**U** = 341° 35′ 45″.883 **B** = 159° 14′ 42″.746
XI	***Tornea*** ***Stuor–oivi***	φ = 65° 49′ 44″.7 φ = 68° 40′ 58″.4	**r** = 166173.82 ± 1.64	**U** = 349° 48′ 49″.509 **B** = 168° 30′ 41″.974
XII	***Stuor–oivi*** ***Fuglenaes***	φ = 68° 40′ 58″.4 φ = 70° 40′ 11″.3	**r** = 115206.77 ± 1.77	**U** = 8° 41′ 22″.854 **B** = 189° 33′ 00″.791

Actually, Struve's final computations came to 12 *"distances between the parallels"* of the principal points, in other words, to lengths of 12 sections of a common meridian arc; they were derived by way of transforming the data of table 1 with spheroid geodesy formulae. However, those *"particular"* meridian arcs were not considered in [Kaptüg, 2007] since azimuths (that is, orientation data) are excluded in this form of presentation. For the aimed comparison the basic table 1 was taken, i.e. Struve's closing geodetic lines ("geodesics") of the 12 arc triangulation segments.

Table 2 presents the other set of data for the aimed comparison: the real accuracy values of *modern geodetic coordinates* of the 13 principal points on "the Struve arc". In five cases (Nos. 7, 8, 10, 11, 13) coordinate values have been reported by surveying agencies of the participating states; in the other five (Nos. 1, 5, 6, 9, 12) – computed with submitted coordinate values of *adjacent* points (decimals in No. 9 are not shown as being not permitted for open publication). The four southern segments of "the Struve arc" were excluded from the

Table 2. Modern coordinates of the 13 principal points of the Struve Geodetic Arc, on the GRS80 ellipsoid [Kaptüg, 2007].

	Name of the SGA principal point	Latitude	Longitude	Height, m	
				geoidal	above sea level
1	***Staro–nekrassowka***	45° 19′ 57.13"	28° 55′ 40.27"	31	24
2	***Wodolui***	47 01 22	29 04 16		210
3	***Ssuprunkowzi***	48 45 03	26 47 52		336
4	***Kremenetz***	50 05 45	25 41 47		409
5	***Belin***	52 02 39.07	25 13 03.16	28	147
6	***Nemesch***	54 39 01.19	25 19 00.44	26	212
7	***Jacobstadt***	56 30 04.853	25 51 23.596	21	81
8	***Dorpat***	58 22 43.880	26 43 12.342	19	66
9	***Mäki–päälys***	60 04 27.......	26 58 11.......	15	143
10	***Kilpi–mäki***	62 38 02.501	26 46 04.169	17	199
11	***Tornea***	65 49 47.529	24 09 25.525	20	12
12	***Stuor–oivi***	68 40 56.849	22 44 45.409	26	597
13	***Fuglenaes***	70 40 11.998	23 39 48.243	25	15

comparison since necessary details pertaining to Nos. 2, 3 and 4 were not advised; their available coordinate values are not precise. The modern geodetic data should strongly relate to *the SGA authentic points*; thus, a detailed study of the history of use of all the 13 principal arc stations was made, being based on primary sources only: contemporary manuscripts (archive documents) and first publications. Then, the problems of the proper reference surface and of metrical representation of the Struve length values [Kaptüg, 2000] were investigated; various possibilities of comparison and acceptable limits of the factors affecting computation accuracy were looked into.

5. COMPARISON

The modern coordinate values Nos. 5 to 13 served as input data to conduct standard computations with spheroid geodesy formulas. The results are presented in tables 3 and 4.

Table 3. Comparison of lengths and azimuths computed from the data of 1857 and 2007

Computation variants: var. 2 (closing geodesics on sea level surface)
var. 3 (closing chords with their ends on sea level surface)

SGA segments	End points of the SGA segments	Input data epoch	Lengths of the lines, m		Spheroid azimuths of the lines	
			var. 2	*var. 3*	*var. 2* (°. ′. ″)	*var. 3* (″)
V	***Belin***	**1857**	290114.1	089.1	181.20.48.7	48.7
	Nemesch ↑	2007	290121.4	096.4	43.2	43.2
		max.error	± 1.6	± 1.6		
		difference	**– 7.3**	**– 7.3**	**+ 5.5**	**+ 5.5**
VI	***Nemesch***	**1857**	208867.8	858.4	189.36.16.2	16.2
	Jacobstadt ↑	2007	208873.5	864.2	13.8	13.8
		max.error	± 1.1	± 1.1		
		difference	**– 5.7**	**– 5.8**	**+ 2.4**	**+ 2.4**
VII	***Jacobstadt***	**1857**	215426.7	416.4	194.17.39.6	39.6
	Dorpat ↑	2007	215424.4	414.2	38.0	38.0
		difference	**+ 2.3**	**+ 2.2**	**+ 1.6**	**+ 1.6**
VIII	***Dorpat*** ↓	»	189386.8	379.9	4.12.37.3	37.3
	Mäki–päälys		189388.5	381.6	35.6	35.6
			– 1.7	**– 1.7**	**+ 1.7**	**+ 1.7**
IX	***Mäki–päälys*** ↓	»	285467.6	443.9	357.55.09.3	09.3
	Kilpi–mäki		285467.6	443.9	09.2	09.2
			– 0.0	**– 0.0**	**+ 0.1**	**+ 0.1**
X	***Kilpi–mäki*** ↓	»	378050.2	995.0	341.35.45.9	45.8
	Tornea		378050.6	995.4	45.0	44.9
			– 0.4	**– 0.4**	**+ 0.9**	**+ 0.9**
XI	***Tornea*** ↓	»	323882.8	848.2	349.48.49.5	49.5
	Stuor–oivi		323892.7	858.0	41.5	41.4
			– 9.9	**– 9.8**	**+ 8.0**	**+ 8.1**
XII	***Stuor–oivi*** ↓	»	224545.0	533.4	8.41.22.8	22.9
	Fuglenaes		224556.4	544.8	09.9	09.9
			– 11.4	**– 11.4**	**+ 12.9**	**+ 13.0**

The deduced numeric differences convince in equal accuracies of the 2nd and 3rd variants of computation, as well as in absence of computation lapses or remarkable distortions within the processing due to adopted presumptions or not counted factors.

The tables 3 and 4 demonstrate *different apparent divergence* of the 1857 results from the comparable modern quantities. To what extent this can be associated with supposed *residual errors* of the historical surveys of *"the Struve arc"*? A thorough analysis came to the determinations that are summarized below. Hypotheses about *natural displacements* of the SGA principal points over the past 160 ~ 190 years are, of course, mentioned; however, what

Table 4. Length differences of the 1857 and 2007 closing geodesics depending on the two alternative values of the conversion factor.

SGA segments	End points of the SGA segments	Length differences (*Struve's value minus modern value*), metres and ppm	
		1 toise = 1.9490605 m	1 toise = 1.949067 m
V	***Belin*** ***Nemesch***	– 7.3 m , – 25 ppm max.error ± 5 ppm	– 6.3 m , – 22 ppm max.error ± 5 ppm
VI	***Nemesch*** ***Jacobstadt***	– 5.7 – 27 max.error ± 5	– 5.0 – 24 max.error ± 5
VII	***Jacobstadt*** ***Dorpat***	+ 2.3 + 11	+ 2.9 + 13
VIII	***Dorpat*** ***Mäki–päälys***	– 1.7 – 9	– 1.1 – 6
	part *Halljall – Mäki-päälys*	*– 2.0 – 24*	*– 1.7 – 21*
IX	part *Mäki-päälys – Svartvira*	*– 0.6 – 20*	*– 0.5 – 16*
	Mäki–päälys ***Kilpi–mäki***	– 0.0 – 0	+ 1.0 + 3
X	***Kilpi–mäki*** ***Tornea***	– 0.4 – 1	+ 0.8 + 2
XI	***Tornea*** ***Stuor–oivi***	– 9.9 – 31	– 8.8 – 27
XII	***Stuor–oivi*** ***Fuglenaes***	– 11.4 – 51	– 10.7 – 48

is today known about this subject in the geographic area involved makes one reject supposing that they could be responsible for the large divergence.

5.1 Tenner's "Lithuanian arc" (segments V–VI).

The 1857 length values deduced by Struve were to a considerable extent affected by his later decision (after 15 years have gone) to revise the Tenner reference length standard (*normal sajen*) that caused its reduction by 9 ppm; some further reduction of Tenner's results came from mathematical adjustment procedures undertaken by Struve. Reference to the first accounts by Tenner and Struve, as well as the recent GPS–remeasure of one of the Tenner "Lithuanian" baselines comes to the firm conclusion that supposed errors of the geodetic transfer of Tenner's length standard to the "Lithuanian" geodesics are approximately *50% less* than the length discrepancies presented in the tables 3 and 4. This conclusion does not depend on whether Tenner's standard has become reduced with time, or has not; it is irrespective at all of *sajen*'s basic relation to the Struve *normal toise*. In other words, *supposed errors of the* *field works carried out by Tenner's military surveyors are significantly smaller than the revealed differences: in terms of length approximately by half.*

On the other hand, it is likely that *some common negative scale error* still is left hidden in Tenner's lengths although there is no direct evidence of that; the supposition comes from the circumstance that the "corrected by half" divergence of the Tenner lengths from the modern values is not negligible. There are still differences in the compared azimuths too; these were not explored within this research due to complexity of the task.

5.2 Struve's "Baltic arc" (segments VII–VIII).

There is revealed relatively small divergence from the modern values in either segment, however, the length differences have opposite signs. At the ends of all "Baltic" segments measured by Struve himself one can see "jumping" length differences and their signs. As to the boundary of segments VI and VII, the apparent "jump" seems to be a trace of the historical junction in 1828 of the "Lithuanian" and "Baltic" meridian arcs measured by Tenner and Struve under very differing geographic and technological circumstances. It is worth mentioning that the supposed excess in length of Struve's VIIth geodesic fits with *excessive* (by 5 ~ 13 ppm) *lengths* of three Struve's triangulation sides that remain in southern Latvia near ***Jacobstadt***; hence, there occurs a supposition that *Struve's measurement technique may have left some positive residual errors.* Contrary to that, the length of the Struve segment VIII compared with his southerly neighbour shows a "jump" to the *negative side*, however, this time the picture cannot be explained definitively. The case may be ascribed to just a small negative accidental error in the northernmost triangle; it may also be attributed to *physical divergence* of the segment end points due to extra *activities of local geodynamic forces*. Local earthquakes were recorded in the Gulf of Finland more than once, and geologists point at relatively fresh rifts recorded around Gogland island where basic point ***Mäki–päälys*** is located.

5.3 Struve's "Finland arc" (segments IX–X).

These segments were measured during 14 years; geometrically, they have relatively longer stretch and unfavourable deviations from the measured lines. Struve estimated the angular accuracy of the "Finland arc" triangles to be *three times lower* than that of the "Baltic arc" triangles. In spite of this each of the "Finland" closing geodesics demonstrates a very small difference in length and azimuth with the comparable modern quantities. Such coincidences with the modern data defy definite explanation. Contrary to that, it is reliably established that several remaining triangle sides in both parts of the "Finland arc" do have *submetre shortages* in their length values (by 13 ~ 20 ppm).

5.4 "Lapland" (Selander's) and "Finmarken" (Hansteen's) arc segments XI–XII.

Relatively big differences of the 1857 results for these segments from the modern situation cannot be attributed to any geodynamical effects and should be mainly explained by *residual errors due to technical aspects and external circumstances of geodetic measurements* in "Scandinavian" arc segments. Triangles of either segment were observed from ground level with the instruments standing on their tripods without means of controlling *stability of the stands*. A special emphasis must be made on *extreme working conditions* beyond the Arctic Circle (arc segment XII); here Norwegians had to use the light, small universal instrument with 10″ accuracy of reading. It would be also plausible to assume that both "Scandinavian" segments had, additionally, a *common negative scale error,* partly due to a hidden systematic error while measuring two key baselines (with the same new rod device), partly due to a considerably multistage structure of the base extension nets.

5.5 Struve's estimation of probable errors of the 1857 lengths.

Having compared data in the tables 1 and 4 and taking into account the above conclusion of a smaller–by–half linear discrepancy of the "Lithuanian" geodesics, it is clear that Struve's estimates of the probable errors generally fit values of linear discrepancies in the eight examined segments of the meridian arc *if the criterion of the maximum possible error* (calculated as three probable or two mean square errors) is applied. A slight violation of the criteria can be seen only in the segment XII.

5.6 Results of the 1994 first GPS– measurement on the Struve arc.

The 1994 GPS–measurement of the Struve arc northern vector ***Mäki–päälys – Fuglenaes*** (1189 km) revealed that the appropriate line computed with Struve's input data had a remarkable "shortage" of its length (by c. 20 m) and its azimuth was by 4″ too high [Kaptüg et al. 1996]. The undertaken research confirms that deduction; the bottom lines of table 3 show evidence that the "shortage" and exceeding azimuth are essential features of the "Scandinavian" parts of "the Struve arc" triangulation where the triangle measurements, due to various reasons, were less perfect than in other parts of the meridian triangulation.

CONCLUSION

The historical significance of *"the Struve Geodetic Arc"* was successively recognized in the 19th , 20th and – as a World Heritage – in this 21st centuries.

Distinguished European geodesists more than once stated *heterogeneity* among various parts of the extended continental triangulation chain of the Russo–Scandinavian meridian arc. Of course, it would be a surprise to find a *perfect uniformity* in the measurements which took 40 years and passed through the 3000–kilometre extent under particular historical conditions, with those geographic, political, technological and bureaucratic distinctions that existed while measurement operations were being slowly advanced by the participants of that unprecedented venture.

Indeed, calculations made within the undertaken research convincingly *prove* some heterogeneity of the five parts of "the Struve arc" that manifests itself in *irregular differences* of their "geometry" as of 1857 from the comparable modern values. Magnitude of linear discrepancies in various parts of "the Struve arc" rises in the following order:

Finland ↗ *Baltic* ↗ *Lithuanian* ↗ *Lapland* ↗ *Finmarken* ,

owing, partially, to residual errors due to *technical aspects* of the measurements. Considering *lengths*, the corresponding residual errors due to the mentioned aspects can be presented with the following *generalized* estimates for the same sequence of "the Struve arc" parts:

(?) ↗ *+ 10 ppm* ↗ *– 15 ppm* ↗ *– 30 ppm* ↗ *– 50 ppm .*

Yet a close look reveals an impact of unknown factors in this succession of uneven residual errors; the most important of them is *an objective (external) inequality of opportunities* to carry out accurate geodetic measurements within the specific environment of particular geographic areas, political and administrative systems of the first half of the 19th century. Leaders of the meridian measurement works: Struve, Tenner, Selander and Hansteen developed, without any doubt, the *optimal* measurement techniques valid for the specific conditions they were aware of; the first three of them used to participate themselves in field works. As a result, all the works completed under their supervision formed an integral, without exceptions, meridional triangulation chain embracing *equally valuable* parts of the world greatest "Russo–Scandinavian" meridian arc measurement.

Sufficient uniformity of the historical measurements was reached thanks to 40–year long field works and most laborious mathematical processing; this quality ensured further extensive use of the results in Earth sciences and surveying practice.

The finished research proves a remarkable level of analytical and mathematical work done by F.G.W.Struve who was the principal worker in research, computation and control of the meridian arc measured elements. The results obtained will hopefully promote better comprehension of the geodetic contents of the World Heritage *"The Struve Geodetic Arc"*, as well as further efforts in preserving its 13 principal stations.

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BIOGRAPHICAL NOTES

Born in 1947. Graduated in astronomy at the Faculty of Mathematics and Mechanics of the Leningrad University. Experience in satellite and astronomical observations, management. Headed several volunteer expeditions aimed at preservation of artifacts of significant geodetic surveys. In 2003–2004 charged with compilation of the national documents for the FIG–UNESCO project "Struve Geodetic Arc". Since 2004 on elected Secretary to the Board of the St. Petersburg Society for Surveying & Mapping. Member of the Russian Geographical Society. Some 50 publications on remarkable geodetic surveys and artifacts in Russia.

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