Spherical and Planar Approach in Determination of Local Geoid: Case Study in Trabzon/Turkey

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Key Words: Gravity, Geoid, FFT, Stokes-Kernel

SUMMARY

Geoid determination is the process of calculation of the length of the ellipsoidal normal (geoid undulation) between the geoid surface and the reference ellipsoid. Various methods are used in determination of the geoid undulations. The solution, that considers a global geopotential model (GM), gravity anomalies (Δg), and topographic effects, is used to determine the gravimetric geoid undulation.

The remove-restore technique is a combination of the spherical harmonic and Stoke's formulation. The long wavelength effects from a geopotential model and short wavelength effects from the topography are mathematically removed from the observed gravity anomalies in this technique. The Stoke's formulation of the residual parts of the gravity anomalies yields the medium wavelength of the geoid height. The geoidal height of a point is determined by restoring the long and short wavelength components. If the area for determining local geoid is chosen small and is considered as planar, it can be divided into M by N grids while distances Δx and Δy are the grid intervals. The geoid undulations can be calculated from Fast Fourier Transform (FFT) solutions of the Kernel functions of the gridded gravity anomalies and distances.

In this study, a 39-point GPS network has been established and the data of this network has been used in order to determine the local geoid covering Trabzon province with remove-restore and FFT techniques. The coordinates of the network points have been transformed into ITRF 96 with epoch 1998.0. The gravity measurements at the network points have been realized based on two reference points (BG_4087 ve BG_4088) whose gravity values are known in the study area. EGM96 global geopotential model is used and the grid intervals are chosen as 1by1 km. The size of the study area is about 12 km by 5 km.

As a consequence, it has been determined that the standard deviation of the spherical approach, Stoke's integral, is 7.4 cm. and the standard deviation of the planar approach, FFT solution, is 8.6 cm in determination process of the geoid undulation. Results obtained from two approaches have been compared and examined. The minimum and maximum absolute values of the differences between geoid undulations obtained from two approaches at a point are 8.5 cm. and 24.3 cm., respectively and the average standard deviation is 5.5 cm.

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1. INTRODUCTION

The solution, which is a combination of a global geopotential model (GM), gravity anomalies (Δg), and topographic effects, is used to determine the gravimetric geoid undulation (Fig. 1.). In this case, the definition of gravimetric geoid undulation can be written as (Sideris, 1994).

$$N = N_{GM} + N_{\Delta g} + N_T \tag{1}$$

where
N_{GM} : Geoid undulation implied by the geopotential model
$N_{\Delta g}$: Contribution of reduced gravity anomalies
N_T : Indirect effect of the topography

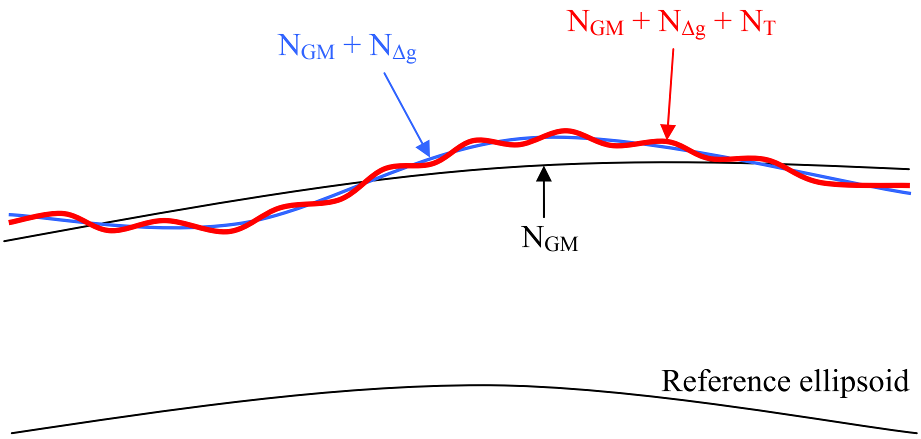

Figure 1. Gravimetric geoid undulation.

1.1 Global Geopotential Model

The contribution of the GM coefficients to the geoid undulation (N_{GM}) of a point is computed by spherical harmonic expansion. Various geopotential models have been developed up to now. Currently, developed geopotential models have the maximum degree and order of 360 (Rapp and Cruz, 1986; Pavlis, 1996). The mostly used geopotential models are EGM96 and OSU91A.
The contribution of geopotential model coefficient at a point on the earth is calculated with spherical harmonic expansions and given by spherical approach on geoid.

$$N_{GM} = R\sum_{n=2}^{n_{max}}\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\sin\varphi) \qquad (2)$$

where

R : Average radius of the earth

$\overline{C}_{nm}$, $\overline{S}_{nm}$: Fully normalized harmonic coefficients of the anomalous potential

$\overline{P}_{nm}$: Fully normalized associated Legendre function

n, n_{max} : The maximum degree and order of expansion of the GM potential solution

φ, λ : geocenteric latitude and longitude of a point.

Similarly, in spherical approach, gravity anomaly on a geoid can be computed from a geopotential model with following equation (Heiskanen and Moritz, 1967).

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$$\Delta g_{GM} = \gamma\sum_{n=2}^{n_{max}}(n-1)\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\sin\varphi) \qquad (3)$$

1.2 Determination of Reduced Gravity Anomalies

The gravity measurements made at physical surface of the earth has to be reduced to geoid surface with a specific gravity reduction. There are several reduction methods such as Bouger reduction, free-air reduction, Helmert's second method of condensation reduction (Martinec vd., 1993), isostatic reduction and residual terrain model reduction.

Usually, free-air anomalies at sea level are used in the Stokes' equation taking into account the masses through the Helmert's second method of condensation reduction. In second method of Helmert's condensation the topographic masses of volume density ρ above the geoid are shifted and condensed to a surface layer of the density (ρxH). Gravity anomaly which is determined by the second method of Helmet's condensation is different from sum of free-air anomaly Δg_{FA} and amount of terrain correction c. (Wicheincharoen, 1982).

$$\Delta g = \Delta g_{FA} + c \qquad (4)$$

This type anomaly is generally called Faye anomaly and the most convenient for the calculation of the geoid undulation. Faye anomaly also contains second indirect effect $\delta\Delta g$ on gravity. Faye anomalies on geoid can be given fully as follows

$$\Delta g_{Faye} = g_p + FA - \gamma + c + \delta\Delta g = \Delta g_{FA} + c + \delta\Delta g \qquad (5)$$

where

g_p : Observed gravity at the calculation points,

FA : Free-air effect on gravity,

c : Conventional terrain correction,

$\delta\Delta g$: Indirect effect on gravity,

Δg_{FA} : Free-air gravity anomaly correction for atmospheric attraction.

Equation (4) is determined based on linear approach of topographic effect (Moritz, 1980).

According to Helmert's second reduction, gravity anomaly Δg is given by

$$\Delta g = \Delta g_{FA} + c + \delta\Delta g - \Delta g_{GM} \tag{6}$$

where

Δg_{GM} : Gravity anomaly on geoid calculated by spherical harmonic expansion at spherical approach.

According to Heiskanen ve Moritz (1967), terrain correction at point (x_i, y_j) is

$$c(i,j) = -G\iint_E \int_{h_{ij}}^{h_{uv}} \frac{\rho(x,y,z)(h_{ij}-z)}{r^3(x_i - x, y_j - y, h_{ij} - z)} dxdydz \tag{7}$$

where

G : Newton's gravitational constant,

$\rho(x,y,z)$: Topographic condensation of a reference point,

h_{ij} : topographic height at point (i,j)

E : Area of integration,

$$r(x,y,z) = \sqrt{x^2 + y^2 + z^2} \tag{8}$$

Equation (7) can be written as follows in case of using a gridded digital terrain model and taken condensation constant.

$$c(i,j) = -G\rho \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \int_{x_n - \Delta x/2}^{x_n + \Delta x/2} \int_{y_m - \Delta y/2}^{y_m + \Delta y/2} \int_{h_{ij}}^{h_{nm}} \frac{h_{ij} - z}{r^3(x_i - x, y_j - y, h_{ij} - z)} dxdydz \tag{9}$$

or similarly,

$$c(i,j) = -G\rho \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \int_{x_n - \Delta x/2}^{x_n + \Delta x/2} \int_{y_m - \Delta y/2}^{y_m + \Delta y/2} \left(\frac{1}{r(x_i - x, y_j - y, 0)} - \frac{1}{r(x_i - x, y_j - y, h_{ij} - h_{nm})} \right) dxdy \tag{10}$$

The different types of c(i,j) can be obtain by assumption of different condensation of topographic masses. Generally, two types of condensation models that are known mass prism model and mass line model are used. These two models are shown in figure 2a and figure 2b.

If mass of prism is condensated mathematically along its vertical axis, topography in prism is presented by a line which gives mass line topographic model figure 2b.

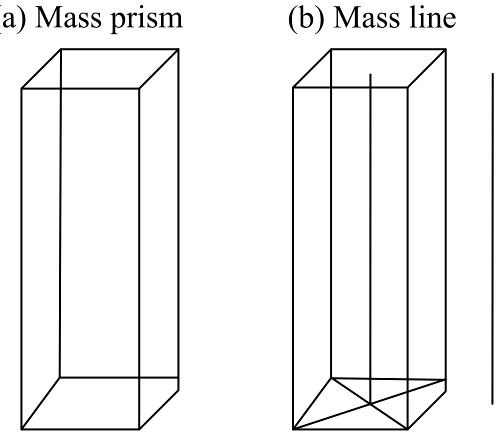

Figure 2. Two different topographic illustrations

With the mass prism topographic model the expression for the terrain c(i,j) is obtained as

$$c(i,j) = G\rho \sum_{n=0}^{N-1}\sum_{m=0}^{M-1}\left[x\ln(y + r(x,y,z)) + y\ln(x + r(x,y,z)) - z\arctan\frac{xy}{zr(x,y,z)} \right] \Bigg|_{x_i-(x_n-\Delta x/2}^{x_i-(x_n+\Delta x/2)} \Bigg|_{y_j-(y_m-\Delta y/2)}^{y_j-(y_m+\Delta y/2)} \Bigg|_{h_{ij}-h_{nm}}^{0} \tag{11}$$

When the mass in prism is condensated along a line, equation (10) can be expressed basically as follows (Yang, 1999).

$$c(i,j) = -G\rho\Delta x\Delta y \sum_{n=0}^{N-1}\sum_{m=0}^{M-1}\left[\frac{1}{r(x_i - x, y_j - y, 0)} - \frac{1}{r(x_i - x, y_j - y, h_{ij} - h_{nm})} \right] \tag{12}$$

1.3 Geoid Undulation and Stokes-Kernel Function on Spherical Approach

In order to determine the aid of reduced gravity anomalies to geoid undulation, Stokes equation is given by

$$N_{\Delta g} = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g(\varphi,\lambda) S(\psi) d\sigma \tag{13}$$

where
σ : Integration sphere
φ ve λ : Latitude and longitude of a data point

γ : Normal gravity
Δg : residual gravity anomaly
$S(\psi)$: Stokes-Kernel function
ψ : difference between data point and measurement point.

In practice gravity data are only effective in limited point areas, equation (13) can be rewritten as equation (14) for gravity anomaly data on sphere (Li and Sideris, 1994).

$$N_{\Delta g} = \frac{R}{4\pi\gamma} \sum_{\varphi=\varphi_1}^{\varphi_B} \sum_{\lambda=\lambda_1}^{\lambda_L} \Delta g(\varphi,\lambda) S(\psi) \cos \Delta\varphi \Delta\lambda \tag{14}$$

where
$\Delta\varphi$, $\Delta\lambda$: Grid intervals at latitude and longitude
L, B : The number of meridians and parallels and studying area in a block

Stokes-Kernel function is given by

$$S(\psi) = \frac{1}{\sin\frac{\psi}{2}} - 4 - 6\sin\frac{\psi}{2} + 10\sin^2\left(\frac{\psi}{2}\right) - \left[3 - 6\sin^2\left(\frac{\psi}{2}\right)\right] \ln\left[\sin\frac{\psi}{2} + \sin^2\left(\frac{\psi}{2}\right)\right] \tag{15}$$

where

$$\sin^2\left(\frac{\psi}{2}\right) = \sin^2\left(\frac{\varphi_p - \varphi}{2}\right) + \sin^2\left(\frac{\lambda_p - \lambda}{2}\right) \cos\varphi_p \cos\varphi \tag{16}$$

1.4 Geoid Undulation and FFT on Planar Approach

Fast Fourier Transform (FFT) is an algorithm for calculation of Fourier transform of discretely gridded data. Due to the suitability to manage data the spectral domain, by the use of the some of the properties of the Fourier transform, its application is mostly used for numerical solutions in physical geodesy, usually in planar approximation. The 2D continuous Fourier transform (CFT) is given as follows (Schwarz et al., 1990).

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) e^{-i2\pi(ux+vy)} dxdy = F[g(x,y)] \tag{17}$$

Here G, is called spectrum of the function g(x,y); u and v are are spatial frequencies in the directions of x and y respectively; i is the imaginary number ($i = \sqrt{-1}$).

The function g(x,y) can be expressed in the space domain by an inverse operation to its Fourier transform by

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}G(u,v)e^{i2\pi(ux+vy)}dudv=F^{-1}[G(u,v)] \tag{18}$$

where, F^{-1} is 2 Dimensional Fourier inverse operator.

In practice our data are only given at discrete points and are of limited extent. Therefore we need a formulation for discrete case. If we estimate of the spectrum for a function on a finite interval, Formulation can be expressed with data given the interval (-X/2≤x≤X/2, -Y/2≤y≤Y/2) as follows (Schwarz et al., 1990).

$$G_F(u,v)=\int_{-X/2}^{X/2}\int_{-Y/2}^{Y/2}g_F(x,y)e^{-i2\pi(ux+vy)}dxdy \tag{19}$$

If we now consider the data to represent a periodic process, the spectrum becomes discrete and the corresponding discrete Fourier transform for discrete gridded data, in the directions x and y, can be approximated by transforming the integrals in equations (17) and (18) into the respective summations as follows

$$G(m\Delta u,n\Delta v)=\Delta x\Delta y\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}g(k\Delta x,l\Delta y)e^{-i2\pi\left(\frac{mk}{M}+\frac{nl}{N}\right)}=DFT[g] \tag{20}$$

m = 0,1,….,M-1,   n = 0,1,….,N-1

$$g(k\Delta x,l\Delta y)=\Delta u\Delta v\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}G(m\Delta u,n\Delta v)e^{-i2\pi\left(\frac{mk}{M}+\frac{nl}{N}\right)}=DFT^{-1}[G] \tag{21}$$

k = 0,1,….,M-1,   l = 0,1,….,N-1

where, $\Delta u=\frac{1}{M\Delta x}$, $\Delta v=\frac{1}{N\Delta y}$, M and N are number of data points in the direction of x and y respectively.

Equations (20) and (21) are base for FFT algorithm in order to evaluate DFT (Schwarz et al., 1990). If the area for determining local geoid is chosen small and is considered as planar, Stokes formulation can be expressed approximately as follows

$$N(x_p,y_p)=\frac{1}{2\pi\gamma}\iint_E \Delta g(x,y)l_N(x_p,y_p,x,y)dxdy \tag{22}$$

where

$$l_N(x_p, y_p, x, y) = \frac{1}{\sqrt{(x_p - x)^2 + (y_p - y)^2}} \quad (23)$$

l_N is given as above equation and it is called approximated planar kernel. If the are is divided into M×N elements with grid interval Δx, Δy, using the gridded gravity anomalies the geoid undulation at point (k, l) can be computed by

$$N(k,l) = \frac{1}{2\pi\gamma}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\Delta g(x_i, y_j) l_N(x_k - x_i, y_l - y_j)\Delta x \Delta y \quad (24)$$

where, l_N is

$$l_N(k-i, j-l) = \begin{cases} (2\pi)^{-1}\left[(x_k - x_i)^2 + (y_l - y_j)^2\right]^{-1/2}, & (x_k \neq x_i \text{ veya } y_l \neq y_j) \\ 0, & (x_k = x_i \text{ ve } y_l = y_j) \end{cases} \quad (25)$$

In frequency domain equation (24) can be expressed by

$$N(x_k, y_l) = \frac{1}{2\pi\gamma} F^{-1}\{\Delta G(u_m, v_n) L_N(u_m, v_n)\} \quad (26)$$

In equation (26) $\Delta G(u_m,v_n)$ and $L_N(u_m,v_n)$, are Fast Fourier Transform of the Δg and l_N as shown below equations (Sideris, 1994).

$$\Delta G(u_m, v_n) = F\{\Delta g(x_k, y_l)\} = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1}\Delta g(x_k, y_l) e^{-i2\pi(mk/M + nl/N)}\Delta x \Delta y \quad (27)$$

$$L_N(u_m, v_n) = F\{l_N(x_k, y_l)\} = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} l_N(x_k, y_l) e^{-i2\pi(mk/M + nl/N)}\Delta x \Delta y \quad (28)$$

1.5 Indirect Effect of the Topography

Indirect effect of Helmert’s condensation reduction on geoid at planar approach can be given as follows (Sideris, 1990).

$$N_T(x_p, y_p) = \frac{\pi G}{\gamma}\rho(x_p, y_p) h^2(x_p, y_p) - \frac{G}{6\gamma}\iint_E \frac{\rho(x_p, y_p)\left[h^3(x, y) - h^3(x_p, y_p)\right]}{s^3} dxdy \quad (29)$$

If the area is gridded by M×N, equation (30) can be easily used instead of equation (29) (Sevilla, 2004).

$$N_T(x_p, y_p) = \frac{\pi G\rho}{\gamma} h^2(x_p, y_p) + \frac{G\rho\Delta x\Delta y}{6\gamma} h^3(x_p, y_p) \sum_{x=x_1}^{x_M} \sum_{y=y_1}^{y=y_N} \frac{1}{s^3}$$
$$- \frac{G\rho\Delta x\Delta y}{6\gamma} \sum_{x=x_1}^{x_M} \sum_{y=y_1}^{y_N} \frac{1}{s^3} h^3(x, y) \qquad (30)$$

2. APPLICATION

In this study, a 39-point GPS network (Figure 3.) has been established and the data of this network has been used in order to determine the local geoid covering province of Trabzon in Turkey. Studying area is generally mountainous. The baselines are taken approximately 1 km. in the network. GPS observations were made using 2 Ashtech Z-Xtreme and 3 Ashtech Z-Surveyor GPS receivers. Observations were completed at 28 sessions. Occupation time in session was taken 45 minutes. Observations were processed using GeoGenius2000 software. In the baseline processing, precision were obtained for the baselines 2.9 mm horizontally and 6.5 mm vertically. In the network adjustment, precision were obtained for the points 5.3 mm horizontally and 6.9 mm vertically. The coordinates of the network points were transferred into ITRF 96 with epoch 1988.0. Gravity measurements of the points were made relatively using Worden Gravity Meter No: 801 Model. Two reference points (BG-4087 and BG-4088) whose gravity values are known were used in relative gravity measurement.

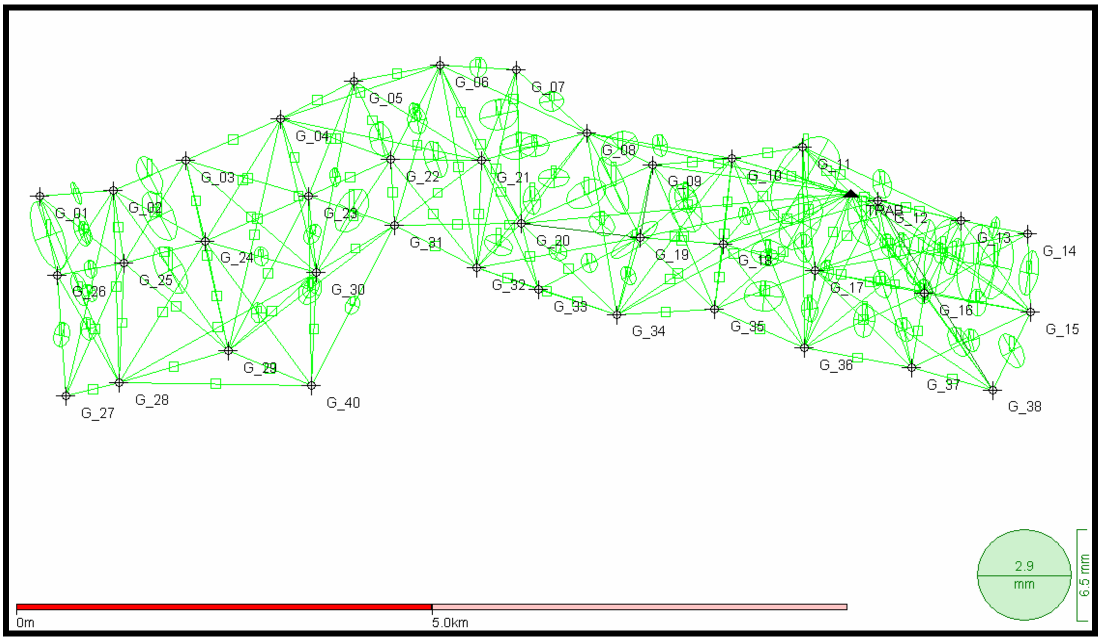

Figure 3. GPS network

Normal gravity values of the points on the ellipsoid are computed by following equations.

$$\gamma = \frac{\gamma_e(1 + k\sin^2\varphi)}{\sqrt{1 - e^2\sin^2\varphi}} \quad ; \quad k = \frac{b\gamma_p}{a\gamma_e} - 1 \qquad (31)$$

where

a : semi major axis of the ellipsoid,
b : semi minor axis of the ellipsoid,
e : eccentricity,
γ_e : Normal gravity at equator,
γ_p : normal gravity at poles.

Grid intervals were taken 1000m in both directions (Δx, Δy). The computations were made based on EGM96 geopotential model using 5×12 data points. MATLAB programming was used in the computations. As a consequence, reduced gravity effects for spherical and planar approaches, combine solution of geoid undulations, and the relation between geoid undulations are given in Table 1. Maximum and minimum value of geoid undulations and their standard deviations are given in Table 2. In these tables, N_s and N_p denote combine values of geoid undulations for spherical and planar approaches respectively.

Table 1. Computation results for geoid undulations at points for spherical and planar approaches and their differences.

Point no.	Spherical Approach		Planar Approach		Difference	Point no.	Spherical Approach		Planar Approach		Difference
	$N_{\Delta g}$ (m)	N_s (m)	$N_{\Delta g}$ (m)	N_p (m)	N_k-N_d		$N_{\Delta g}$ (m)	N_s (m)	$N_{\Delta g}$ (m)	N_p (m)	N_s-N_p
G_01	0.0490	24.8953	0.0124	24.8587	0.0366	G_21	0.0526	24.8036	0.0174	24.7684	0.0352
G_02	0.0573	24.8883	0.0139	24.8449	0.0434	G_22	0.0045	24.7683	0.0015	24.7653	0.0030
G_03	0.0184	24.8113	0.0105	24.8034	0.0079	G_23	0.0585	24.8689	0.0139	24.8243	0.0446
G_04	0.0536	24.7939	0.0157	24.7560	0.0379	G_24	0.0050	24.8724	0.0015	24.8690	0.0035
G_05	0.0295	24.7238	0.0175	24.7118	0.0120	G_25	-0.0221	24.8789	-0.0034	24.8975	-0.0186
G_06	0.0251	24.6920	0.0218	24.6887	0.0033	G_26	-0.0034	24.9173	-0.0005	24.9202	-0.0029
G_07	0.0397	24.6997	0.0244	24.6843	0.0153	G_27	-0.1097	24.9288	-0.0165	25.0220	-0.0932
G_08	0.0467	24.7559	0.0242	24.7334	0.0225	G_28	-0.2617	24.7572	-0.0186	25.0003	-0.2431
G_09	0.0312	24.7610	0.0219	24.7518	0.0093	G_29	-0.0386	24.9300	-0.0081	24.9605	-0.0305
G_10	0.0144	24.7267	0.0199	24.7321	-0.0055	G_30	-0.0695	24.8152	-0.0142	24.8705	-0.0553
G_11	-0.0049	24.6863	0.0178	24.7090	-0.0227	G_31	0.0476	24.8730	0.0122	24.8375	0.0355
G_12	-0.0076	24.7234	-0.0034	24.7276	-0.0042	G_32	0.0270	24.8806	0.0081	24.8617	0.0189
G_13	-0.0213	24.7158	-0.0115	24.7257	-0.0099	G_33	0.0058	24.8739	0.0061	24.8742	-0.0004
G_14	-0.0092	24.7313	-0.0135	24.7270	0.0043	G_34	0.0903	24.9699	0.0055	24.8850	0.0849
G_15	-0.0339	24.7804	-0.0381	24.7763	0.0042	G_35	0.0049	24.8653	0.0109	24.8713	-0.0060
G_16	-0.0616	24.7514	-0.0526	24.7604	-0.0090	G_36	0.0095	24.8950	0.0127	24.8982	-0.0032
G_17	0.0316	24.8402	0.0231	24.8317	0.0085	G_37	-0.1097	24.7792	-0.0693	24.8195	-0.0403
G_18	0.0383	24.8337	0.0209	24.8163	0.0174	G_38	-0.1139	24.7821	-0.0907	24.8052	-0.0231
G_19	0.0271	24.8290	0.0110	24.8129	0.0161	G_40	-0.1737	24.8231	-0.0276	24.9691	-0.1460
G_20	0.0016	24.8102	0.0092	24.8178	-0.0075						

Table 2. Maximum and minimum value of Geoid undulations and their standard deviations

Geoid undulation	Max. value (m)	Min. value (m)	Standard deviation (cm)

$N_{\Delta g}$ (Spherical)	0.090	-0.262	6.9
$N_{\Delta g}$ (Planar)	0.024	-0.091	2.6
$N_{spherical}$	24.970	24.686	7.4
N_{planar}	25.022	24.684	8.6
Difference ($N_{spherical}$–N_{Planar})	-0.243	0.085	5.5

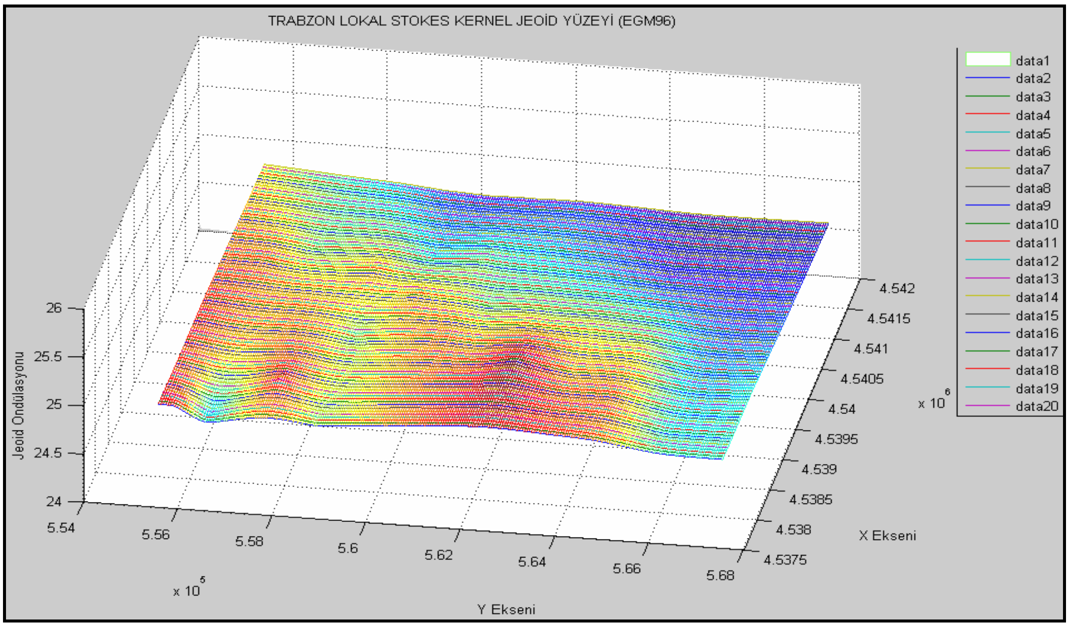

Figure 4. Gravimetric geoid surface for spherical approach

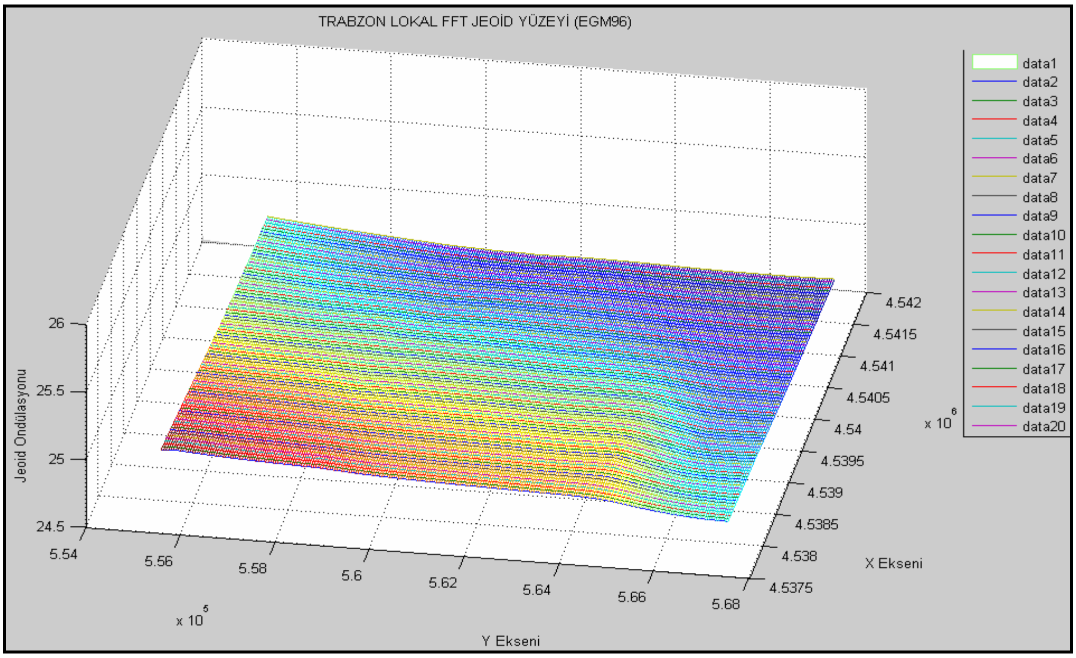

Figure 5. Gravimetric geoid surface for planar approach

3. CONCLUSION

As a result of this study, the minimum and maximum absolute values of the differences between geoid undulations obtained from two approaches at a point 8.5 cm and 24.3 cm respectively. The examination of gravity effect to geoid undulations showed that variation between maximum and minimum values and standard deviation are smaller on planar approach with respect to spherical approach. However, obtained geoid undulations from combined solution showed that variation between maximum and minimum values and standard deviation are smaller on spherical approach with respect to planar approach. Herein, when examining the results we should take into account of the working area is small and mountainous.

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