AN ALTERNATIVE APPROACH IN METROLOGICAL NETWORK DEFORMATION ANALYSIS EMPLOYING KINEMATIC AND ADAPTIVE METHODS

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Abstract: The LHC (Large Hadron Collider) project includes the construction of four large physics experiments, which will study particle collisions. The particle detectors that are made up of many parts need to be precisely positioned with respect to the accelerator beam line. The metrological networks to accomplish this task are in difficult configurations. Control and thus reliability degrade as detector installation progresses. Additionally, deformations of the structural parts of the experimental caverns are to be expected which will affect the networks and need to be monitored closely. As part of the installation process the network will be regularly measured in parts, including different types of measurements, but complete network measurements will be very rare. Good network configuration and reliability at early stages of the installation process need to be fully taken advantage of. This implies a good preliminary estimation of possible point movements. This knowledge could be utilized in later stages when network configuration and reliability will degrade. Another source of information about the network is empirical knowledge how the network points might deform depending on their location. This information could also be considered into the network calculations in order to support its solution. To treat these problems with classical deformation methods based on epoch-to-epoch congruency comparison poses several problems in application. A practical and easier to handle solution is the implementation of a kinematic model by an adaptive Kalman filter. In this paper we will present a special implementation of an adaptive Kalman Filter which is able to take full advantage of any measurement occurring in the cavern network context and to manage and maintain accuracy and reliability demands for the detector installation and operation.

1. Introduction

The European Organization for Nuclear Research (CERN – Centre Européen pour la Recherche Nucléaire) is currently concentrating on the construction and installation of the new particle accelerator LHC (Large Hadron Collider). In order to study elementary particle physics four large high energy physics experiments have been installed along the accelerator beam line, each with different physics objectives and detector designs. These experiments have been installed partly in already existent underground caverns, but for the two largest experiments ATLAS (A Toroidal LHC ApparatuS) and CMS (Compact Muon Solenoid) extensive excavation work had been necessary.

A particle detector consists of various detector systems which again comprise many parts. All these parts have to be positioned accurately with respect to each other and the total of the detector accurately positioned with respect to the accelerator beam line. Geometrical networks have been installed in order to support and control this process. Although the caverns are of considerable dimensions (e.g. ATLAS 53 x 30 x 35m, cavern floor ~92m below ground, see Fig. 1), space for survey work is restricted and limitations apply to network configurations.

Underground openings in such a dimension cannot be expected to be stable over a long time period. Deformation will occur and will affect the geometrical relation between the LHC tunnel and the experimental caverns and thus between accelerator beam line and detector.

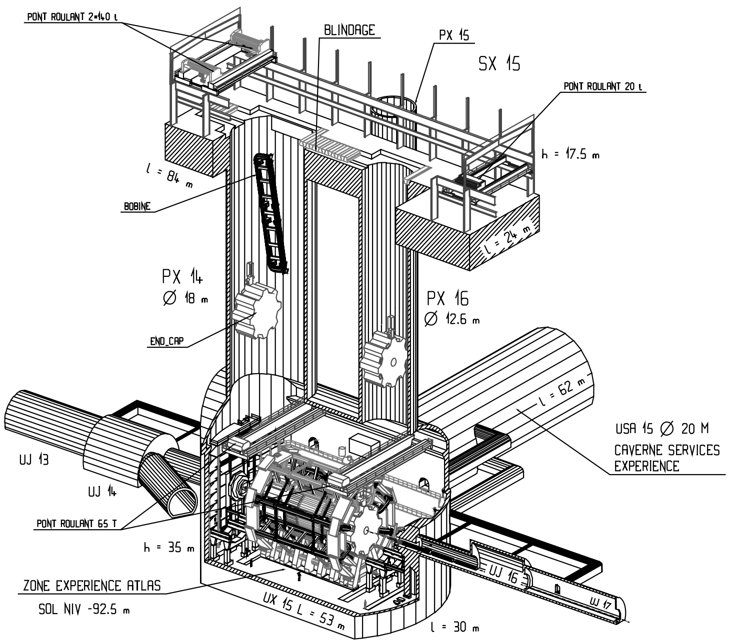

Figure 1: ATLAS detector installation at LHC Point 1, [1]

It is inevitable that the network configuration will degrade with progressing installation. In the beginning a cavern is empty and only few limitations on measurement configurations apply. Once infrastructure and detector installation commence many obstacles restrict sightings, thus less possible measurements can be carried out, which means poorer control of the measurements. Reliability in the network decreases which is unacceptable in order to continuously support and control detector installation accurately. It is obvious that the configuration of measurements in such a network will rarely be the same even twice.

Thus two major problems for the survey work to support and control detector installation in the cavern have to be faced: One is the deformation of the cavern directly affecting the network's stability. Secondly, geometrical limitations increase as installation progresses causing the network's reliability to degrade. Nevertheless, it is necessary to maintain a high level of reliability. In order to solve these two problems in one common approach an algorithm employing an adaptive Kalman Filter applied to a kinematic 3D network is proposed.

2. Algorithm

The Kalman Filter (KF) as a recursive minimum mean-square error (MMSE) estimation algorithm has been extensively used ever since its first formulation. This is also true for applications in the geodetic field, most prominently in the field of navigation. Its application to deformation measurements has been introduced in several publications, e.g. [6]&[3].

2.1. Kinematic network

A network is commonly assumed stable over time and deformation occurring only locally and/or exceptionally. For the task at hand we interpret a metrological network as a time-variable system. This system is subject to generally small, creeping changes and only exceptionally to large and sudden changes. Causes of these deformations are in general not sufficiently well known to derive a clear relationship. Instead of detecting the actual amount of deformation and make a distinct separation between changing and stable points, the main objective in this approach has to be to maintain the necessary level of reliability in the network. All points are considered as changing and the amount by which they change is estimated.

In the context of deformation models the approach chosen here can be categorized as a *descriptive model*, as causes for deformation or points movements are generally not known and no cause-response model can be established, [7]. Point variations are modelled as function of time, which gives a *kinematic model.*

To model the system of a deforming, three-dimensional geodetic network we set up system equations for an unforced, uniformly accelerated motion. Higher order terms of motion are considered as system noise and are thus not explicitly modeled, [6]. The differential equation reads:

$$\ddot{\mathbf{x}}(t) = \mathbf{w}(t); \qquad \ddot{\mathbf{x}}(t) - \mathbf{w}(t) = \mathbf{0}\text{, where } \mathbf{x}(t) \text{ is position and } \mathbf{w}(t) \text{ is random noise.} \tag{1}$$

This yields after integration over $\Delta t = (t_i - t_{i-1})$:

$$\mathbf{x}(t_i) = \mathbf{x}(t_{i-1}) + (t_i - t_{i-1}) \cdot \dot{\mathbf{x}}(t_{i-1}) + \frac{1}{2} \cdot (t_i - t_{i-1})^2 \cdot \ddot{\mathbf{x}}(t_{i-1}) + \mathbf{w}(t_{i-1}) \,. \tag{2}$$

From this expression we derive in the following the system model for the treatment of geodetic network measurements in a Kalman Filtering process.

2.2. Kalman Filter for geodetic network

The KF system equation is obtained as:

$$\hat{\mathbf{x}}_{i|i-1} = \mathbf{\Phi}(i, i-1) \cdot \hat{\mathbf{x}}_{i-1|i-1} + \mathbf{\Gamma}(i-1) \cdot \mathbf{w}_{i-1} \,, \tag{3}$$

where $\hat{\mathbf{x}}_{i|i-1}$ represents the state variable estimate at time *i* based on measurement information up to including time *i-1,* whereas $\hat{\mathbf{x}}_{i-1|i-1}$ is the updated state at time *i-1*, i.e. the estimate at time *i-1*, based on measurement data up to including time *i-1*. $\mathbf{\Phi}(i, i-1)$ is the transition matrix, $\mathbf{\Gamma}(i-1)$ is the noise matrix.

The measurement equation reads:

$$\mathbf{z}_i = \mathbf{H}_i \cdot \mathbf{x}_i + \mathbf{v}_i \,, \tag{4}$$

where $\mathbf{z}_i$ contains the measurement data for time *i*, $\mathbf{H}_i$ represents the linear observation equations and $\mathbf{v}_i$ describes the measurement noise.

The system noise $\mathbf{w}_{i-1}$ and measurement noise $\mathbf{v}_i$ describe model disturbances and noise corruption that affect the system, but also uncertainty about the model. They are characterized as follows:

$$E\{\mathbf{w}_{i-1}\}=0; \qquad E\{\mathbf{w}_{i-1}\cdot\mathbf{w}_{j-1}^T\}=\mathbf{Q}_{i-1}\cdot\delta_{ij}$$
$$E\{\mathbf{v}_i\}=0; \qquad E\{\mathbf{v}_i\cdot\mathbf{v}_j^T\}=\mathbf{R}_i\cdot\delta_{ij} \tag{5}$$
$$E\{\mathbf{v}_i\cdot\mathbf{w}_{j-1}^T\}=\mathbf{0} \text{ for all i, j.}$$

The KF formulation is summarized in Fig. 2, see also [2]. A discrete formulation is chosen as measurement data in geodetic applications are discrete i.e. sampled at different instances in time.

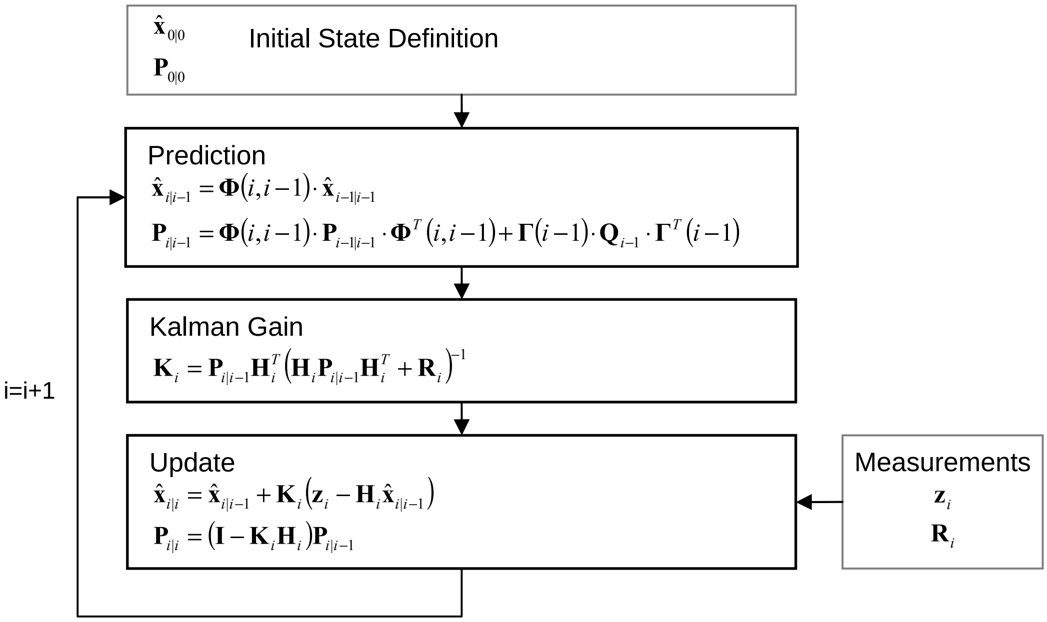

Figure 2: Kalman Filter formulation

2.2.1. Setup KF terms for kinematic 3D network

The state vector $\mathbf{x}$ is to be interpreted as a composite: It contains for each point in the 3D network position coordinate components, velocity coordinate components and acceleration coordinate components. The system transition matrix $\mathbf{\Phi}(i,i-1)$ describes the system model i.e. kinematic point description derived from the original differential equation in (2):

$$\mathbf{\Phi}(i,i-1)=\begin{bmatrix}\mathbf{I} & \Delta t\cdot\mathbf{I} & 0.5\cdot\Delta t^2\cdot\mathbf{I}\\ \mathbf{0} & \mathbf{I} & \Delta t\cdot\mathbf{I}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix} \text{ for } \Delta t=t_i-t_{i-1}\,. \tag{6}$$

For the application of a geodetic network the matrix of linear/ linearized observation equations has only entries for the 'position states':

$$\mathbf{H}_i=\left[\mathbf{H}_{i\_pos} \quad \mathbf{H}_{i\_vel} \quad \mathbf{H}_{i\_acc}\right]=\left[\mathbf{H}_{i\_pos} \quad \mathbf{0} \quad \mathbf{0}\right] \tag{7}$$

It can be shown that although only position states are observed by the measurements, also velocities and accelerations can be estimated in this algorithmic approach, [5].

2.3. Adaptive KF

The KF needs to *adapt* to deformations in the network, which are not known before and which are likely to change over time. The system description includes a certain level of uncertainty, the KF algorithm is thus able to let the estimates adapt within this uncertainty's range. So, if we expect deformation, the system uncertainty should be large enough to let the system adapt to a new situation, on the other hand, if we know that there is no deformation, we want the system uncertainty to be fairly small, not to degrade the already good state estimation. Thus the idea is to set the a priori system uncertainty fairly small and to let the KF determine itself, when system uncertainty has to be increased or not, based on the comparison between new measurement data and the system (i.e. the innovation analysis).

The motivation for any adaptive filter is the need to correctly identify unknown parameters in the stochastic model of the KF. This is done by analyzing the mismatch between the measurement data and the current system description, i.e. the *innovation*. Various adaptive methods exist. Commonly used autocorrelation methods applied to innovations are unlikely to give conclusive results in the application of a geodetic network. This is due to the small and incomplete data sample sizes and thus poor statistics. A less demanding but also less conclusive method is referred to as *stochastic stabilization*: Only the system noise variance is considered unknown (a correct stochastic model for measurement data is imperative). Its difficult direct derivation is compensated by an iterative increase of the noise variance (i.e. variance inflation) until the system has sufficiently adapted to incorporate new information, which 'stabilizes' the filter.

The adaptive filter is based on the innovation representing the mismatch between actual measurements and the best available prediction based on the system model and previous measurement data:

$$\mathbf{d}_i = \left(\mathbf{z}_i - \mathbf{H}_i \hat{\mathbf{x}}_{i|i-1}\right). \tag{8}$$

2.3.1. The global model test

To decide when adaption is necessary, the innovation property, [4] is used:

$$E\left\{\mathbf{d}_i \cdot \mathbf{d}_j^T\right\} = 0 \quad \text{for } i \neq j . \tag{9}$$

If the optimal gain $\mathbf{K}_i$ has been found no information is contained in the innovation. Its mean and covariance are:

$$\begin{aligned} &E\{\mathbf{d}_i\} = 0 \\ &E\left\{\mathbf{d}_i \cdot \mathbf{d}_i^T\right\} = \mathbf{H}_i \mathbf{P}_{i|i-1} \mathbf{H}_i^T + \mathbf{R}_i = \mathbf{D}_i \end{aligned} \tag{10}$$

The innovation is considered to be a gaussian distributed variable,

$$\mathbf{d}_i \sim N(\mathbf{0}, \mathbf{D}_i). \tag{11}$$

Thus we can define a test term $\Omega^2_{\mathbf{d}_i}$ under the assumption of no significant innovation

$$\Omega^2_{\mathbf{d}_i} = \frac{\mathbf{d}_i^T \cdot \mathbf{D}_i^{-1} \cdot \mathbf{d}_i}{\sigma_o^2} \sim \chi^2_{1\text{-}\alpha,\mathrm{f_i}} \tag{12}$$

with the probability relationship

$$P\left\{\Omega^2_{\mathbf{d}_i} \leq \chi^2_{1\text{-}\alpha,\mathrm{f_i}} \middle| H_0\right\} = 1 - \alpha . \tag{13}$$

If the null hypothesis of the global model test (13) is rejected with a significance level of α, the observations carry information new to the system, which needs to be changed.

2.3.2. Localization

The test statistic for the global test in (12) is based on innovations, thus in the 'domain' of the measurements. The term to be changed to let the filter adapt is the system noise which describes uncertainties in the system state variables. If the global test indicates a significant mismatch between new observation data and the system we need to localize the source of this mismatch in the domain of the state variables. To achieve this we need to find a test term equivalent to expression (12) but in the state 'domain'.

The innovation is related to the state by the gain matrix, (see update equation in Fig. 2). The term $\mathbf{K}_i \cdot \mathbf{d}_i$ represents an incremental update to the state and has properties equivalent to those of the innovation:

$$\mathbf{v}_{\mathbf{x},i} = \mathbf{K}_i \cdot \mathbf{d}_i \,. \tag{14}$$

The corresponding covariance matrix is given by:

$$\mathbf{P}_{\mathbf{v}_{\mathbf{x},i}} = \mathbf{K}_i \cdot \mathbf{D}_i \cdot \mathbf{K}_i^T \,. \tag{15}$$

If equation (11) is valid it can be likewise said that

$$\mathbf{v}_{\mathbf{x},i} \sim N\left(\mathbf{0}, \mathbf{P}_{\mathbf{v}_{\mathbf{x},i}}\right). \tag{16}$$

Thus the test statistic from expression (12) can be reformulated into:

$$\Omega^2_{\mathbf{v}_{\mathbf{x},i}} = \frac{\mathbf{v}^T_{\mathbf{x},i} \cdot \mathbf{P}^{-1}_{\mathbf{v}_{\mathbf{x},i}} \cdot \mathbf{v}_{\mathbf{x},i}}{\sigma_o^2} \sim \chi^2_{1\text{-}\alpha,\mathrm{h_i}} \,. \tag{17}$$

In order to localize the reason for the mismatch between the system and the measurement data indicated by the global test, state parameter(s) which cause the mismatch have to be identified. The entries of the state and thus of $\mathbf{v}_{\mathbf{x},i}$ can be grouped into a vector corresponding to the 3D point *j* they describe, i.e. variables for position, velocity and acceleration for point *j*. Thus test statistics $\Omega^2_{\mathbf{v}_{\mathbf{x},i\_j}}$ for each individual point *j* can be derived, [5] and it has to be decided, how to proceed in the algorithm in order to let the filter adapt to the new information. In case only one point would be suspected to have caused the mismatch between the system and measurement data, a classical outlier detection approach could be chosen, by changing the system noise information for this point and iterate the calculation. But as in classical outlier detection this method is prone to error if more than one item is suspected to have caused the significant test term. Similar to an approach presented in [8] we propose a slightly modified method which is considered to be more robust.

For each point *j* we derive the test term $T_{i\_j} = \dfrac{\Omega^2_{\mathbf{v}_{\mathbf{x},i\_j}}}{\chi^2_{1\text{-}\alpha,\mathrm{h_{i\_j}}}}$. (18)

Stochastic stabilization is achieved by applying the following changes to the system noise error definition:

$$\begin{array}{ll} T_{i\_j} > 1 & \mathbf{q}_{i-1} = \mathbf{q}_{i-1} \cdot \exp T_{i\_j} \\ T_{i\_j} \le 1 & \mathbf{q}_{i-1} = \mathbf{q}_{i-1} \end{array} \tag{19}$$

which implies that all points with a significant test statistic are down-weighted. As the process is iterated until no significant mismatch between the measurement data and the system is indicated, points experiencing large position changes can change their state estimates correspondingly, but likewise the uncertainty in the state estimate increases.

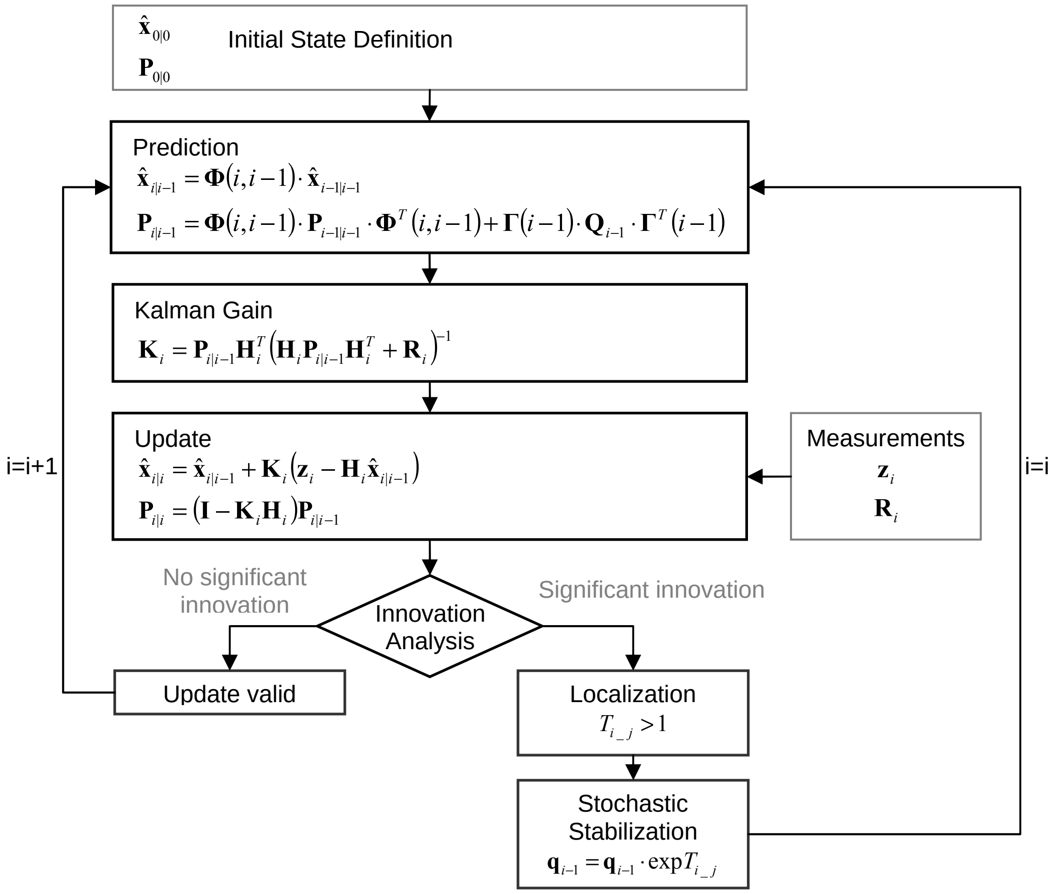

Figure 3: Adaptive Kalman Filter – Stochastic Stabilization

2.4. Including additional information in KF

The result of a KF is a function of observation data, thus as much observation information as possible should be included. In deformation applications often some kind of empiric knowledge about the probable deformation characteristics exists, (e.g. points on the cavern floor are expected to move mostly in vertical direction). But very often this information can not be quantified or modelled in a classical sense (cause-response model). Including this kind of information with suitable stochastic information can support the KF estimation process, particularly in difficult observation configurations as in the example of an experiment cavern.

Formulating conditions or constraints on the estimation parameters to be included in the parameter adjustment is a commonly used technique in geodetic applications. The conditions are cast into so called 'virtual' observations. A simple example of such a constraint applied to a deformation adjustment in KF formulation is a *soft datum* definition: As observations serve the known coordinates of datum points. Applied to the kinematic model implemented in the KF the soft datum constraint can be imposed on the position states, but also constraints on the velocity and acceleration states can be imposed, e.g. $\mathbf{x}_{vel} = \mathbf{x}_{acc} = \mathbf{0}$. Relationships between point movements can be also modelled in this way, if, for example, points are expected to deform in a similar way.

3. Application and results

A small simulation example shall serve to present first results for the adaptive KF for a 3D geodetic network in kinematic setup. The simulation represents a simplified version of the ATLAS design network. Distances and distribution of the network and object points are represented by less points: 12 network points and 8 object points, see Fig. 4. The measurement configuration is different in all 10 measurement epochs: the network is measured only in parts, object points are introduced starting in epoch 6, some points are in some epochs only partly determined, etc. A simplified deformation is applied by a rise of the four cavern floor network points. Other network and object points remain stable.

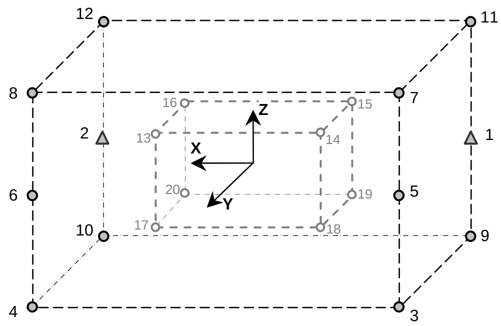

Figure 4: Simulated network layout

3.1. Comparison adaptive KF with single-epoch least squares

In order to see the performance of the adaptive KF in kinematic setup the results for some network and object points are now compared with results obtained by single epoch least squares adjustment results, not considering a kinematic setup. In the following figures the true, simulated positions are indicated by small squares, wheras the respective estimates are indicated by small '+'s. The respective 1-σ error ellipses are plotted in their original size. 2D plots in the X/Y coordinate plane are presented.

3.1.1. Network point, observed in 9 of 10 epochs, simulated movement in z-direction

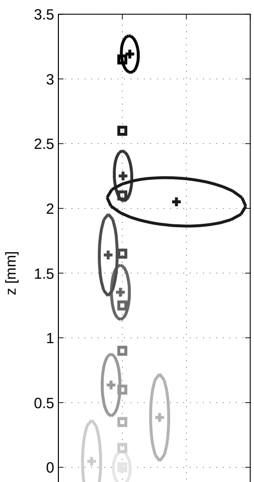

Figure 5: P9 – single-epoch LS, static

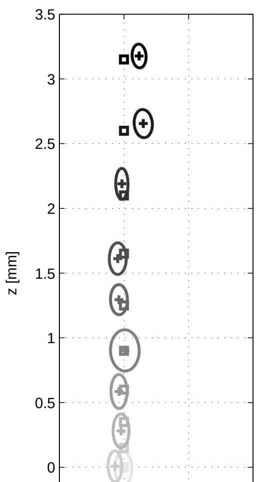

Figure 6: P9 - adaptive KF, kinematic

Comparing Fig. 5 and Fig. 6 the superiority of the second approach is seen in more characteristics than the absolute difference in the coordinate results: If no measurements are available in one epoch (i.e. epoch 5) no estimate can be made in the single epoch case. The KF estimate is nevertheless accurate, due to a good prior estimation of the point movement. The error ellipse is slightly inflated, which correctly expresses the systems elevated uncertainty about the estimate. In case the point is poorly determined (epoch 8), the adaptive KF is less affected than single epoch least squares.

3.1.2. Object point, observed in 3 epochs

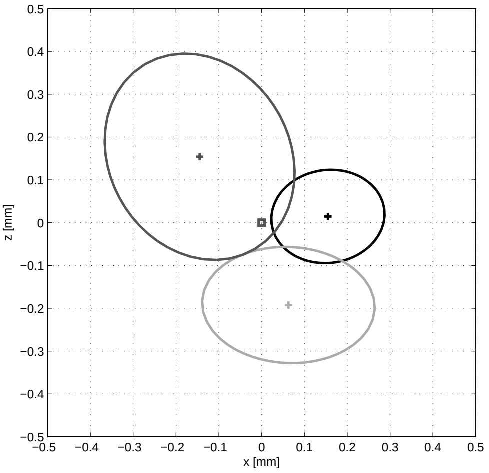

Figure 7: P19 – single-epoch LS, static

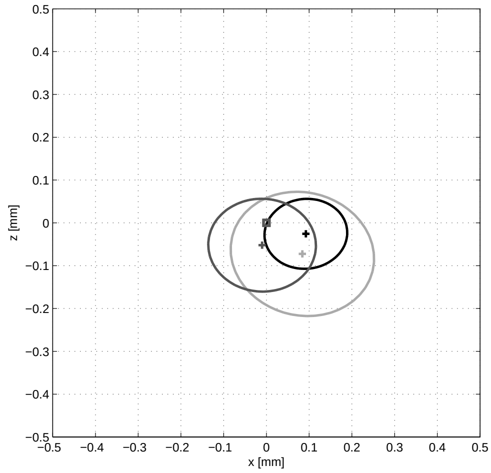

Figure 8: P19 - adaptive KF, kinematic

In the case of an object point (representing the detector parts to be installed), the profit of the kinematic becomes evident: As depicted in Fig. 7 and Fig. 8, a single position becomes more accurately determined, when a kinematic and adaptive KF approach is applied.

3.2. Including additional information

If we include additional empirical information about the expected deformation the performance of the adaptive KF can even be improved. For the simulated example presented above, we can formulate the expectation that the deformation of the cavern floor points will be in vertical direction (i.e. in Z- coordinate) only.

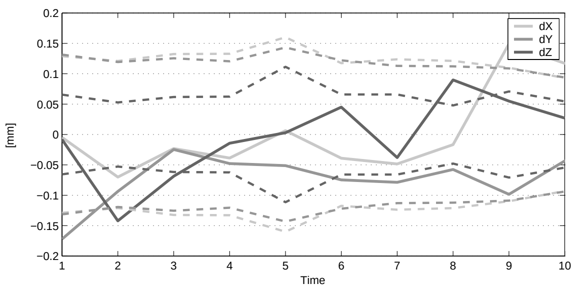

Figure 9: P9 – no additional information

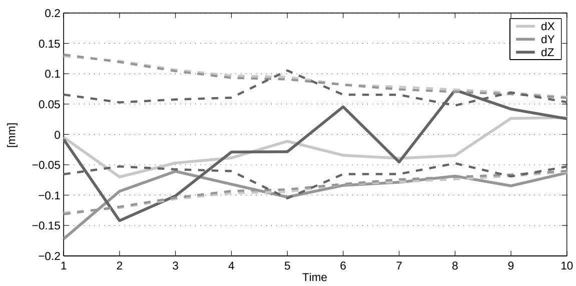

Figure 10: P9 – additional information included

Obviously the additional information has its most prominent effect on the points it contains information about - in the case here on the cavern floor points. Nevertheless, the improvement can be seen overall in a better network reliability. Coordinate differences to the true position are shown in Fig. 9 and Fig. 10. The overall accuracy of the point can be improved by including additional information.

3.3. Real Data

Real data from ATLAS cavern have recently become available and are extensively analyzed in [5].

4. Conclusion

We presented a method of coping with a deforming 3D geodetic network in changing configurations. The developed algorithm employs a kinematic setup for network points and estimation is carried out by an adaptive Kalman Filter. The implemented algorithm is capable of taking full advantage of any measurement and additional information to maintain network accuracy and reliability while providing complete deformation analysis at the same time. The motivation for this method is the special task of positioning particle detectors at CERN. Advantages of the chosen approach have been illustrated using a simulated example of the ATLAS cavern.

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