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# FROM GEODETIC MONITORING TO DEFORMATION TENSORS AND THEIR RELIABILITY

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## Abstract

When a specific area has to be monitored, several techniques are available. One of them is to survey a geodetic network at different epochs. Each survey provides the positions of the points at a given time. The first results are obviously the displacements, from which, in a second step, one can process the deformation field. Deformation tensors are a very efficient instrument to perform this calculation and can be very helpful to analyse the behaviour of the studied area. Indeed, three independent measures - dilatation, shear and twist - can be derived. These three parameters describe the way and the magnitude of the deformation.
Besides, these deformation parameters should be characterized by an indicator of precision since the uncertainties in the observations and measurements propagate through out the computation and then lead to uncertainties in the results. This reliability mainly depends on measurements standard-deviations and network geometry. However, the calculation itself of these deformation parameters contributes also to the error propagation. Therefore, two methods have been considered.
The first method is used when the calculation of the deformation parameters can be linearized. Variances-covariances matrix propagation rules can be applied so that standard-deviations on the deformation parameters are processed following all mathematical rules and can be determined in a formal way.
If the calculation can not be linearized, a second method has been studied. Thus, a new property is introduced to characterize the network: its deformability. This is calculated by the way of virtual deformations created by an artificial change of the observations using Monte Carlo method. This deformability is actually the slightest deformation the network is able to detect and thus, one can check if the observation of the a priori designed network can reveal the expected deformation. Finally, the deformation reliability at a point is defined by the comparison between the network deformability at this point and the real deformation that has been measured.
This work deals with a 2D problematic. Several studies conducted at the Department of Geodesy and Levelling of the Institut Géographique National - France, have been used as practical trials and will be presented.

## 1. Introduction

When a structure is monitored by the way of topometric or geodetic networks, the points displacements are determined with respect to reference points that are supposed to be located outside the area of interest. However what is of importance for the structure is the internal deformation that the structure has to face with. That’s why it has been found interesting to analyze the displacements one can get with a surveying monitoring network in terms of deformations. And the defomation tensors are an efficient tool for such an analysis. But, as in any interpretation of data processed from observations that are affected with uncertainties, it becomes compulsory to have an idea of the confidence level we can have on these computed tensors. That’s what the study presented in this article deals with.
First, deformation tensors are briefly presented. Then, the notions of deformability of a monitoring network and deformation significance are defined, before presenting a way of computing them and some practical examples. Finally, perspectives are exposed since this study is still being developed.

## 2. Deformation tensors

It is convenient to present briefly the notion of deformation tensors and the hypotheses in which this study is leading.

### *2.1 Definition of deformation tensors*

Deformation tensors gather most of the information of the behaviour of a displacement vectorial field. Given a local displacement field around a given point M(x,y),

$$\delta(x,y) = \begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} ,$$

the deformation tensor E(x,y) is given by the gradient of this displacement field at this point:

$$E(x,y) = \begin{bmatrix} e_{ux}(x,y) & e_{uy}(x,y) \\ e_{vx}(x,y) & e_{vy}(x,y) \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x}(x,y) & \frac{\partial u}{\partial y}(x,y) \\ \frac{\partial v}{\partial x}(x,y) & \frac{\partial v}{\partial y}(x,y) \end{bmatrix} .$$

By calculating the gradient of the displacement field, one gets rid of the reference that has been used for the calculation of the displacement field since only relative variations are considered. However, the interpretaion of such a tensor is not obvious but its decomposition may help extracting some characteristic elements.

### *2.2 Decomposition of deformation tensors*

Several decompositions of deformation tensors do exist. In this study, the one that is used is the decomposition in symmetrical and antisymmetrical parts. This decomposition leads to

$$E = \begin{bmatrix} e_{ux} & e_{uy} \\ e_{vx} & e_{vy} \end{bmatrix} = S + A ,$$

où $S = \begin{bmatrix} e_{ux} & \frac{e_{uy}+e_{vx}}{2} \\ \frac{e_{uy}+e_{vx}}{2} & e_{vy} \end{bmatrix} = \begin{bmatrix} \varepsilon_{ux} & \varepsilon_{uy} \\ \varepsilon_{vx} & \varepsilon_{vy} \end{bmatrix}$ et $A = \begin{bmatrix} 0 & \frac{e_{uy}-e_{vx}}{2} \\ -\frac{e_{uy}-e_{vx}}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_z \\ \omega_z & 0 \end{bmatrix}$ .

S represents the symetric tensor of deformation, also called Cauchy's tensor. A is the antisymetric deformation tensor.
This decomposition is used since it is assumed that the hypothesis of small deformations is checked. Indeed, the gradient processing is acting as a filter in order to consider the internal evolution of the studied area. A global translation of the whole area is cancelled out. However a global rotation lets appear quadratic terms in the deformation tensor. The hypothesis of small deformation implies that this quantity is negligeable and leads to the preceding linearization and the following interpretation.

### *2.3 Deformation primitives*

Deformation primitives are quantities defined by the deformation tensor. They represent the deformation in a more meaningful way. These parameters are usual in material characteristics and mechanics.
From the symetric tensor S, one can qualify the principal constraints. S is diagonalisable since it's a real symetric matrix. It's usually represented by an ellipse with the following parameters:

- the eigenvectors of S determine the direction of principal axes,

- the eigenvalues of S determine the length of the semi-axes.

The principal constraints are the eigenvalues. A negative constraint is a contraction while a positive one is an expansion. Dilatation (simple or total) and shift (pure, simple or total) are defined from S.

The antisymetric tensor A can be understood as a rotation $\omega_z$. This rotation is composed by a rigid rotation Ω which affects the whole area and a differential rotation or twist $\delta\omega_z$.

Depending on the use and the goal of the monitoring network, there can be chosen three independent primitives in order to characterize the deformation of the area: one for the dilatation, one for the shift, and one for the twist. Furthermore, dealing with the significance of deformation implies that it is possible to evaluate the deformation. Therefore, a norm of deformation tensor should be defined. The idea is then to process the reliability for each deformation primitive instead of defining the reliability of a deformation tensor.

## 3. Deformability of a network and deformations significance

### *3.1 Definition*

A geodetic network has to be understood as a structure where points and stations are linked to each other by the observations. Random errors caused by instruments, centering, environment affect these observations. These random errors are quantified and define the standard deviation associated to each observation what is used to weight the observation during the Least Squares adjustment of the network. Therefore, the coordinates of the points are determined with a processed precision.

In order to have an information about the deformation such a network is able to detect, the first idea is to make the observations vary in their confidence interval, then to compute the new point coordinates and the displacement. Finally it results the deformation quantified by the three deformation primitves for each of those artifical displacements. It then becomes possible to determine how the network answer to any observation changes with respect to its standard deviation. That's what one can call the deformability of the network. Consequently, any deformation whose magnitude is less than the deformability of the network, can not be considered as significant.

In this study, the deformation primitives that have been chosen are positive scalars, growing with the deformation amplitude :

- Total dilatation : $\lambda = \sqrt{e_{ux}^2 + e_{vy}^2 + \frac{1}{2}\left(e_{uy} + e_{vx}\right)^2}$
- Total shift : $\gamma = \frac{1}{2}\sqrt{\left(e_{ux} - e_{vy}\right)^2 + \left(e_{uy} + e_{vx}\right)^2}$
- Differential rotation : $\delta\omega = \omega_z - \Omega = -\frac{e_{uy} - e_{vx}}{2} - \Omega$

### *3.2 Computation of Deformability*

Several sets of artificial observations are created by using the Monte Carlo method whose parameters are the standard deviations of the observations. Then coordinates are adjusted by Least Squares. In the following one obtains the virtual displacements as differences between the original set of coordinates and the artifical one. From this virtual displacement, virtual deformation tensors are processed and finally virtual deformation primitives.

Given a number sim of simulations, the mean and the standard deviation of all the virtual deformation primitives, these artificial observations lead to the following definition of deformability:

$$\begin{cases} \bar{\lambda} = \dfrac{1}{sim} \displaystyle\sum_{aléa=1}^{sim} \lambda_{aléa} \\ \bar{\gamma} = \dfrac{1}{sim} \displaystyle\sum_{aléa=1}^{sim} \gamma_{aléa} \\ \delta\bar{\omega} = \dfrac{1}{sim} \displaystyle\sum_{aléa=1}^{sim} \left|\delta\omega_{aléa}\right| \end{cases} \quad \text{and} \quad \begin{cases} \sigma_{\lambda} = \sqrt{\dfrac{1}{sim-1} \displaystyle\sum_{aléa=1}^{sim} \left(\lambda_{aléa} - \bar{\lambda}\right)^2} \\ \sigma_{\gamma} = \sqrt{\dfrac{1}{sim-1} \displaystyle\sum_{aléa=1}^{sim} \left(\gamma_{aléa} - \bar{\gamma}\right)^2} \\ \sigma_{\omega} = \sqrt{\dfrac{1}{sim-1} \displaystyle\sum_{aléa=1}^{sim} \left(\left|\delta\omega_{aléa}\right| - \delta\bar{\omega}\right)^2} \end{cases}$$

finally,

$$\boxed{\begin{aligned} \lambda_{def} &= \lambda \pm 2.57\sigma_{\lambda} \\ \gamma_{def} &= \gamma + 2.57\sigma_{\gamma} \\ \delta\omega_{def} &= \delta\omega + 2.57\sigma_{\omega} \end{aligned}}$$

This definition lets 99% of virtual deformation to be taken into account assuming that the set is following a normal distribution.

This deformability can be useful when designing the network. Indeed, simulations can be done with theoretical measurements token as the original observations and by applying a Monte Carlo noise to get the artificial observations and finally the virtual deformation.

## *3.3 Deformation significance*

Deformation significance has to point out the level of confidence of the deformation primitive values. This confidence level depends on :

- The way deformation tensors are computed
- Uncertainties in the observations
- The Geometry of the network
- The compensation by least squares of the network at a given time
- The deformation primitives

Using the same notation, one can define the degree of significance by the following formulae:

$$\boxed{\begin{aligned} \Sigma_{\lambda} &= \frac{\lambda - \lambda_{def}}{\lambda_{def}} \\ \Sigma_{\gamma} &= \frac{\gamma - \gamma_{def}}{\gamma_{def}} \\ \Sigma_{\omega} &= \frac{\left|\delta\omega\right| - \delta\omega_{def}}{\delta\omega_{def}} \end{aligned}}$$

This degree of significance is negative when the measured deformation is not significant since it has a smaller magnitude than the deformability. If it is positive, the measured deformation is meaningful.

## 4. Processing

### *4.1 Network compensation*

The network is compensated by Least Squares method using a software developed by Y. Egels at IGN. Monte Carlo method has been implemented and all the simulations have been realized with this program. Each set of point displacements between the original position and the artificial one is organized in an output file.

### *4.2 Deformability*

Displacement fields are processed from a set of displacement set by an interpolation on a grid following a bicubic expression in order to assure the continuity and the homogeneity of elastic medium. Once the displacement field computed, the deformation tensors are processed on each element of the grid, then the deformation primitives. Means and standard deviations are then computed in order to get the deformability.
All these computations have been realized with Matlab.

### *4.3 Example*

A geodetic network observed at two different epochs in the same way is presented here as an example. First, the deformability is shown for the primitives of dilatation (Fig. 1) and shift (Fig. 2). The deformability is growing from blue to red : smaller deformations can be detected in the blue area , bigger in the red one. Then the figures Fig. 3 and Fig. 4 show the reliability on the effective deformations that have been measured. Only a deformation in dilatation  between two points in the North East of the studied region is significant.

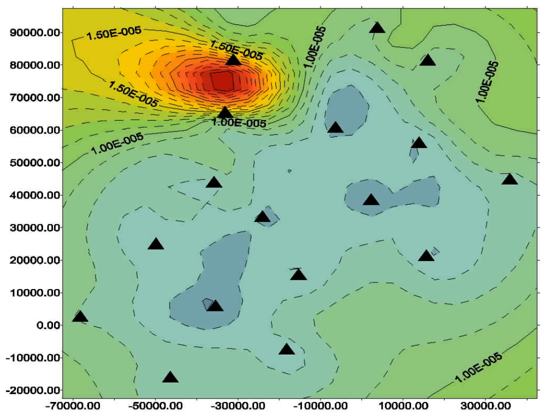


Fig. 1 : Deformability in dilation

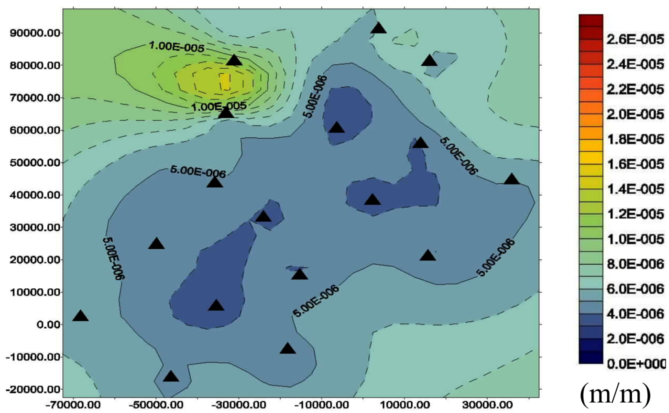


Fig. 2 : Deformability in shift

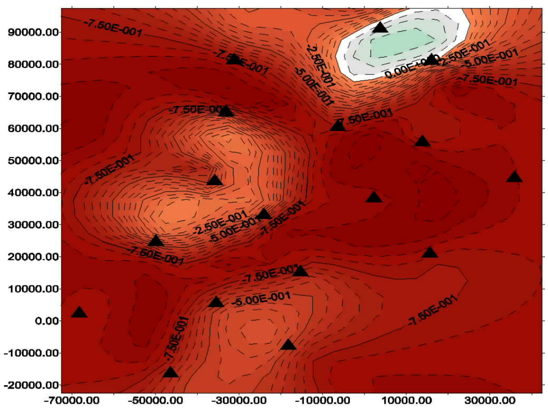


Fig. 3 : Reliability in dilation

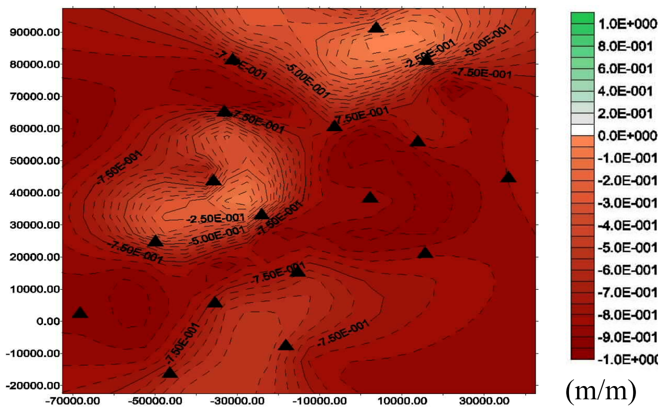


Fig. 4 : Reliability in shift

## 5. Perspectives

### *5.1 Displacement field*

Another way of processing the displacement field is actually being studied by finding the minimal deformation energy using conjugate gradients whereas the displacement on the network points are held fixed to the processed value.
If the characteristics of material resistance (modulus of elasticity for instance) are known, it could be possible to find the eventual structure breaks or terrain fault where the measured deformation is significant and too large to be accepted by the elastic material. Furthermore, the displacement field could be processed considering heterogeneous mediums of elastic materials. This processing could even be applied using Finite Element Method by giving particular properties to each element of the interpolation grid.

### *5.2 Linearization*

Monte Carlo method could be replaced by using variances-covariances matrices and their propagation rules through out linear or linearizable processing.

### *5.3 Extension to 3D*

This study has been made on 2D objects only. Naturally, the extension to 3D objects has to be one of the final goals. Indeed, as least squares compensation of geodetic networks can deal with 3D data, it becomes evident to deal with 3D deformations. However, interpretation of 3D tensors is far more complex and the number of deformation primitives is bigger.

### *5.4 Geotechnical measurement integrations*

The main goal of this study is to conclude from displacements measured by geodetic monitoring networks to deformations. Furthermore geotechnical instruments measure directly deformations and such measurements should be integrated in the study of network deformability.

## 6. Conclusion

This article presents some of the notions that the interpretation of geodetic monitoring networks into deformations has been requiring. Deformability and reliability of deformations had to be defined. As part of these definitions, processing has been set up and applied to real networks to check their validity. However, there are still points to study thoroughly in order to be more efficient and more rigorous.

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