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A KINEMATIC ANALYSIS PROGRAM FOR DEFORMATION MONITORING

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Abstract

This paper introduces a Fortran program called KINDEF for 3D deformation detection via geodetic methods, and describes theory and procedure of it. KINDEF is a kinematic deformation analysis program that uses Kalman-filter technique and performs 3D statistical analysis to inspect the significance of geodetic network point displacements, velocities and acceleration of displacements coming from three repeated GPS surveys of the same network. KINDEF uses Kinematic Single Point Model that is one of the kinematic models. Information about theory and an application of it were given in the article

1. Introduction

Nowadays, deformation surveys have been became one of the most important application areas of geodesy. In deformation monitoring studies, static deformation models are usually used. Static models are sufficient in studies where time is neglected. However, most of the current engineering applications require monitoring of movement behaviors. In such studies, kinematic deformation models determining displacements, velocities and acceleration as dependent on time are preferred.

This paper introduces a Fortran program called KINDEF for 3D deformation detection via geodetic methods, and describes procedures of it. KINDEF is a kinematic deformation analysis program and performs 3D statistical analysis to inspect the significance of geodetic network point displacements, velocities and acceleration of displacements coming from three repeated surveys of the same network. For deformation detection, KINDEF uses Kinematical Single Point Model solved by Kalman-Filter. In this program, movement parameters (displacements, velocities, acceleration) are statistically tested and moved points, velocities and accelerations of moving points are determined. The program was written by Microsoft Fortran Visual Workbench v.1.0 editor being a windows-based and using maximum memory. It has only one-screen facility for representing the results of deformation detection, numerical representation. KINDEF has been used successfully to analyze repeated GPS surveys belonging to a geodetic network in Trabzon province (TURKEY) established for landslide monitoring and control. Test results of the KINDEF were given.

2. Mathematical Theory

Kinematic models allow to estimate the velocity and even the acceleration (by building double differences) of control point movements. Because this is done for every single point, these type of models is called ‘single point’ deformation models. Here the one with the so called ‘Hannover approximation’ is applied. The unknown parameters of a single point deformation model are the velocity and the acceleration of control points. Therefore, a time-dependent function is required to estimate these parameters. The most common approach for this type of model is a quadratic polynomial function

$$x_{k+1} = x_k + a\ (t_{k+1} - t_k) + \frac{1}{2}\ b\ (t_{k+1} - t_k)^2 \tag{1}$$

Where, x_{k+1} is the coordinate vector at time t_{k+1}, x_k the coordinate vector at time t_k,a the velocity vector at time t_k, and b the acceleration vector at time t_k. Eq.(1) can be used to compute both coordinates and parameters of motion, if a sufficient number of observations is available. The Hannover approximation considered here applies three steps to compute the unknown parameters (Zippelt 1986) whereby a minimum of five different observation periods are required. The first step is identical to the static model described before (Leonhard and Niemeier 1986).

Step 1: Static model

$$x_{k+1} = x_k\ . \tag{2a}$$

Among the results are the priori variance of unit weight s_0 and the estimated variance of unit weight m_0. Both are used to calculate the global test statistic T_S which is compared with the α-percentage point q_S of the F-distribution with rang r and f degrees of freedom

$$T_S = \frac{s_0^2}{m_0^2} > q_S \tag{2b}$$

The outcome is used to estimate the velocity vector in step 2.

Step 2: Linear model

$$x_{k+1} = x_k + a\ (t_{k+1} - t_k)\ . \tag{3a}$$

Among the results are the priori variance of unit weight s_0 and the estimated variance of unit weight m_L. Both are used to calculate the global test statistic T_L which is compared with the α-percentage point q_L of the F-distribution with rang r and f degrees of freedom

$$T_L = \frac{s_0^2}{m_g^2} \quad q_L \tag{3b}$$

The outcome is used to estimate the acceleration vector in step 3.

Step 3: Quadratic model

$$x_{k+1} = x_k + a\ (t_{k+1} - t_k) + \frac{1}{2}\ b\ (t_{k+1} - t_k)^2. \tag{4a}$$

Among the results are the priori variance of unit weight s_0 and the estimated variance of unit weight m_Q. Both are used to calculate the global test statistic T_Q which is compared with the α-percentage point q_Q of the F-distribution with rang r and f degrees of freedom

$$T_Q = \frac{s_0^2}{{}^{2}} > q_Q \tag{4b}$$

If there are less than five different observation periods available, the method described before is not applicable. In such cases, a Kalman filtering approach may help as introduced hereafter (Pelzer 1986). A Kalman filter method can be used for regular and irregular point movements. For Eq.(1) a Kalman filter function for the state vector is given by (Pelzer 1987)

$$\bar{y}_{k+1} = \begin{bmatrix} x_{k+1} \\ a_{k+1} \\ b_{k+1} \end{bmatrix} = \begin{bmatrix} I & I(t_{k+1}-t_k) & I\frac{1}{2}(t_{k+1}-t_k)^2 \\ 0 & I & I(t_{k+1}-t_k) \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ a_k \\ b_k \end{bmatrix} + \begin{bmatrix} I\frac{1}{2}(t_{k+1}-t_k)^2 \\ I(t_{k+1}-t_k) \\ I \end{bmatrix} \xi_k \tag{5}$$

$$\bar{y}_{k+1} = R_{k+1,k}\ \hat{y}_k + S_{k+1,k}\xi_k$$

where $R_{k+1,k}$ is the trend or prediction matrix, $S_{k+1,k}$ the noise matrix, $\bar{y}_{k+1}$ the state vector at time t_{k+1} containing trend and noise, and $\hat{y}_k$ the regular state vector at time t_k, and ξ_k the random noise vector at time t_k. The variance-covariance matrix of Eq.(6) reads

$$C_{\bar{y}\bar{y},k+1} = R_{k+1,k}\ C_{\hat{y}\hat{y},k}\ R^T_{k+1,k} + S_{k+1,k}\ C_{\xi\xi,k}\ S^T_{k+1,k} \tag{6}$$

Where, $C_{\hat{y}\hat{y},k}$ is the variance-covariance matrix of $\hat{y}_k$, and $C_{\xi\xi,k}$ is the variance-covariance matrix of ξ_k . Eqs.(5-6) can be combined with the linear Gauss-Markov model as follows

$$\begin{bmatrix} \bar{y}_{k+1} \\ L_{k+1} \end{bmatrix} + \begin{bmatrix} v_{\bar{y},k+1} \\ v_{l,k+1} \end{bmatrix} = \begin{bmatrix} I \\ A_{\bar{y},k+1} \end{bmatrix} \hat{y}_{k+1}\ , \tag{7a}$$

$$C_{\hat{y}\hat{y},k+1} = C_{\bar{y}\bar{y},k+1} \quad K_{k+1} D_{k+1} K^T_{k+1}\ , \tag{7b}$$

where L_{k+1} represents the observation vector at time t_{k+1} and K_{k+1} the consolidation matrix

$$K_{k+1} = Q_{\bar{y}\bar{y},k+1} A^T_y D^{-1}_{k+1}\ . \tag{8}$$

Using the equations above, the innovation vector d_{k+1} and its variance-covariance matrix at time t_{k+1} are given

$$d_{k+1} = L_{k+1} - A_{\bar{y}}\bar{y}_{k+1}$$

$$D_{k+1} = Q_{ll,k+1} + A_{\bar{y},k+1} Q_{\bar{y}\bar{y},k+1} A^T_{\bar{y},k+1} \tag{9}$$

so that the regular state vector at time t_{k+1} can be computed by

$$\hat{y}_{k+1} = \bar{y}_{k+1} + K_{k+1} d_{k+1} \tag{10}$$

Finally, the innovation vector d_{k+1} can be tested by the hypothesis

$$H_0\ :\ E[d_{k+1}] = 0 \tag{11a}$$

against

$$H_1\ :\ E[d_{k+1}] \neq 0 \tag{11b}$$

with the test statistic T

$$T = \frac{d^T_{k+1} D^{-1}_{k+1} d_{k+1}}{n_{k+1} s^2_o} < F_{n_{k+1},f,1-\alpha} \tag{11c}$$

for its statistical significance. If H_0 is true, the computed results are acceptable (Yalçınkaya 1994, 2003).

2. Program Description

Program was designed and written for 3D-GPS network. The starting point of this evaluation is a free-net adjustment for each measuring period resulting in adjusted coordinates, their estimated variance of unit weight, and their estimated variance-covariance matrix. By using the single point static model, the other kinematic models (linear and quadric) are calculated step-by-step as suggested by the Hannover approach using Kalman-filter technique. To decide about the expansion of the model with velocity and acceleration respectively, statistical tests are performed in the program using Equations 2b, 3b and 4b.

Table 1 shows statistical test results of kinematic single point model process. The most proper functional model for the kinematic single point model is shown in Table 1 in decision column. Here, a priori variance (s_0) is computed preliminary adjustment. A posteriori variance (m_0) is computed from the model and a posteriori variance of expanded model (m_g) is computed from the parameters of the expanded model. The test values (T and T_g) are computed with a priori and a posteriori values and compared with the F-distribution table value (q) (confidence level is α=0.05) and it is decided whether the expansion of the model with velocity and acceleration parameters respectively are significant or not. Please see (Koch 1999; Mierlo 1978) for more information about statistical test theory and application.

Table 1. Statistical test results of kinematic single point model

	Linear Model (Position + Velocity)			**Quadric Model** (Position + Velocity+ Acceleration)		DECISION (The Most Suitable Model)
	Global Test	Expanded Model Test		Global Test	Expanded Model Test	
Period	s_0 m_0 T q_f	s_0 m_g T_g q_g	Period	s_0 m_0 T q_f	s_0 m_g T_g q_g	
Nov. 2000 Feb. 2001	0.741 0.765 1.066 1.516	0.765 17.222 560.750 1.669	Nov. 2000 Feb. 2001 May. 2001	0.765 0.839 1.204 1.484	0.839 4.637 30.528 1.613	POSITION + VELOCITY + ACCELERATION

Table 2 shows results computed from three periodic GPS measurements made on November 2000, February 2001 and May 2001. Movement parameters (position (d_x, d_y, d_z), velocity (v_x, v_y, v_z), acceleration (a_x, a_y, a_z)) of this model were computed and results were given in Table 2. Here, every parameter is divided by their square mean error and thus test values (T) are computed. The test values are compared with the t-distribution table value (q) and it is decided whether movement parameters are significant or not using Equation 11c. If parameters have significantly changed, a (+) sign is given in Table 2, otherwise a (-) sign is assigned.

In this paper, in order to do comparison and bring up superiority of kinematic approach, the same problem was solved by a static model (θ^2-criterion). Results were listed for the model in Table 3. Please see (Öztürk, 1978; Yalçınkaya and Tanır 2000) for more information about theory and application of it.

Table 2. Kinematic single point model deformation results

Kinematic Single Point Model Position + Velocity + Acceleration										
If **T > q_t** then **(+) "Stable"** If **T < q_t** then **(-) "Moved"** q_t : t-test value T : test values										
Models	**Linear Model**					**Quadric Model**				
Periods	**November 2000 – February 2001**					**November 2000 – February 2001 – May 2001**				
	q_t = 1.989					q_t = 1.979				
Points	**1**	**2**	**4**	**8**	**9**	**1**	**2**	**4**	**8**	**9**
dx (cm)	-6.10	-0.03	1.20	-0.03	-0.80	-16.09	0.07	1.80	0.09	-3.94
T_{dx}	45.19	0.26	9.03	0.27	5.06	36.64	0.33	7.62	0.37	15.75
Decision	(+)	(-)	(+)	(-)	(+)	(+)	(-)	(+)	(-)	(+)
dy (cm)	5.84	-0.02	0.42	-0.26	0.16	18.47	-0.02	-0.27	-0.52	1.32
T_{dy}	45.87	0.18	3.35	1.86	1.04	26.71	0.08	2.14	1.02	5.12
Decision	(+)	(-)	(+)	(-)	(-)	(+)	(-)	(+)	(-)	(+)
dz (cm)	-2.94	-0.12	1.16	-0.12	-0.23	-6.40	-0.53	1.54	-0.21	-1.82
T_{dz}	20.49	1.13	8.32	0.82	1.34	16.64	1.39	6.47	0.84	6.98
Decision	(+)	(-)	(+)	(-)	(-)	(+)	(-)	(+)	(-)	(+)
v_x (cm/ay)	-2.03	-0.01	0.40	-0.01	-0.27	-1.32	0.02	-0.21	0.02	-0.79
Tv_x	36.43	0.15	7.30	0.07	4.11	12.49	0.35	2.07	0.37	6.32
Decision	(+)	(-)	(+)	(-)	(+)	(+)	(-)	(+)	(-)	(+)
v_y (cm/ay)	1.94	-0.01	0.14	-0.04	-0.06	2.26	0.00	-0.39	-0.04	0.55
Tv_y	33.91	0.11	2.60	0.83	0.91	12.33	0.01	3.96	0.73	4.39
Decision	(+)	(-)	(+)	(-)	(-)	(+)	(-)	(+)	(-)	(+)
v_z (cm/ay)	-0.98	-0.02	0.39	-0.02	-0.76	-0.17	-0.07	-0.27	-0.02	-0.46
Tv_z	16.78	0.67	6.88	0.40	1.09	2.50	1.45	2.50	0.26	3.39
Decision	(+)	(-)	(+)	(-)	(-)	(+)	(-)	(+)	(-)	(+)
a_x (cm/ay^2)						0.12	0.002	-0.104	0.001	-0.089
Ta_x						4.62	0.21	4.12	0.12	2.88
Decision						(+)	(-)	(+)	(-)	(+)
a_y (cm/ay^2)						0.05	0.000	-0.09	-0.001	0.101
Ta_y						2.07	0.06	3.72	0.13	3.29
Decision						(+)	(-)	(+)	(-)	(+)
a_z (cm/ay^2)						0.14	-0.003	-0.110	-0.001	-0.064
Ta_z						5.06	0.39	4.18	0.05	2.92
Decision						(+)	(-)	(+)	(-)	(+)

Table 3. Static (θ^2-criterion) deformation results

Static Model (θ^2-Criterion)										
Periods	**November 2000 – February 2001**					**November 2000 – May 2001**				
Points	**1**	**2**	**4**	**8**	**9**	**1**	**2**	**4**	**8**	**9**
dx (cm)	-5.61	Stable	1.27	Stable	-0.54	-13.82	Stable	2.26	Stable	-3.12
dy (cm)	5.34		0.35		-0.03	16.23		-0.54		1.03
dz (cm)	-2.60		1.22		0.05	-4.84		1.82		-1.36
Decision	**Moved**		**Moved**		**Moved**	**Moved**		**Moved**		**Moved**

3. Discussion

A comparison of the static model using θ^2-criterion and the correspondent value in the Kalman-filter method for kinematic single point model are listed in Tables 2 and 3. These tables show that the result for both model are generally close together and reveal similar characteristic for point movements. When examining results of Table 2 and 3, it can be seen that directions of movement parameters computed with both model are the same. As a result, it can be said that both model results are harmonious. As different from the static model, kinematic single point model provided computation of velocities and accelerations of displacements.

Acceleration parameter have physical meanings. The sign of acceleration is of significant importance to be able to interpret deformations. If acceleration is greater than 0, velocity of deformation increases. If acceleration is less than 0, velocity of deformation decreases. Physical environmental conditions usually determine the sign of acceleration. This evaluation enables a deformation analysis to be made more realistically with respect to physical realities.

As a result, the kinematic single point model solved by Kalman-filter is a statistically sound technique and capable of detecting very small movements. Combining GPS technology with the kinematic approach provides surveyor with a powerful tool for movement detection. Once it is implemented, it is very easy to apply in deformation monitoring.

4. Conclusion

KINDEF is a kinematic deformation analysis program and performs 3-D statistical analysis to inspect the significance of geodetic network point displacements, velocities and acceleration of displacements coming from three repeated surveys of the same network. Program provides estimating movement velocities and accelerations in addition to displacements using Kinematic Single Point model and Kalman-Filter method.

Technical Information

Fortran code of the KINDEF was not given in the paper due to limitation of number of pages. A Fortran Visual Workbench v.1.0 version of the program code will be provided on request from Mualla Yalçınkaya (mualla@ktu.edu.tr) or Temel Bayrak (tbayrak@ktu.edu.tr). They can also provide more detailed theoretical explanations about the program and give an example about use of it.

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