# Introduction to Vertical Reference Frames and Datums

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# Here's were we are on the agenda …

- **09:00 – 10:00 Session 1: Introduction to 3D/Vertical Reference Frames**
  - **1) Introduction to 3D Reference Frames**
    - ***Mr. Nic Donnelly,*** *Land Information New Zealand*
  - **2) Introduction to Vertical Reference Frames and Datums**
    - ***Dr. Dan Roman,*** *NOAA National Geodetic Survey National Oceanic& Atmospheric Administration*
- **10:00 – 10:30 Morning Tea**

# OUTLINE

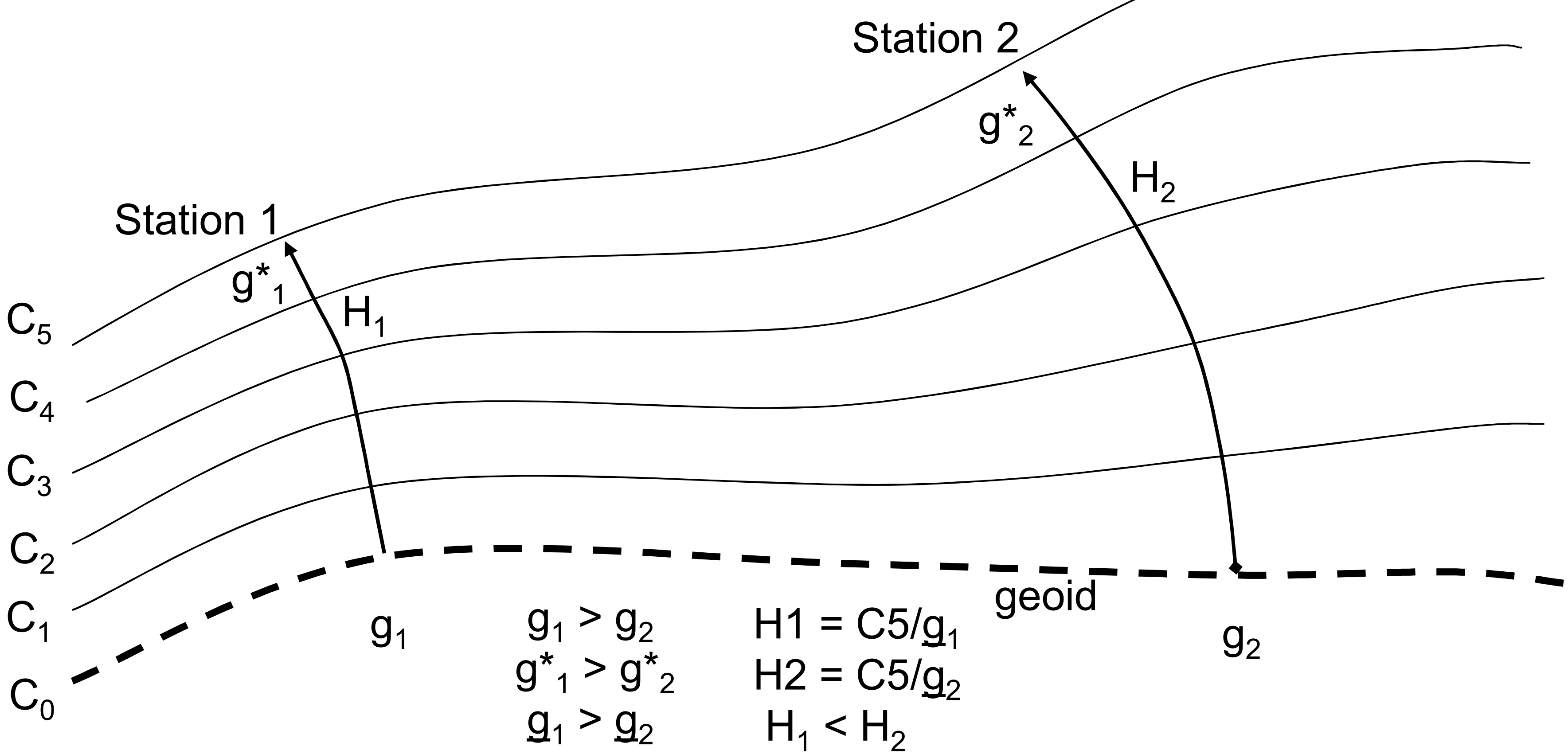
- Basic Definitions
- Why “Mean Sea Level” is not an ideal reference surface
- Problems with an adjustment of a national vertical datum
- Use of a geoid height model instead
- Functions of the Earth’s gravity field are related
- Long vs. short wavelength gravity
- Combination of gravity products
- IHRS

# Basic Definitions

- Geodesy is the science of accurately measuring and understanding three fundamental properties of the Earth: its geometric shape, its gravity field, and its orientation in space, as well as the changes of these properties with time.
- A vertical reference frame or datum is a surface of zero elevation to which heights of various points are referred in order that those heights be in a consistent system. More broadly, a vertical datum is the entire system of the zero elevation surface and methods of determining heights relative to that surface. Over the years, many different types of vertical datums have been used. The most predominant types today are tidal datums and geodetic datums.

# What types of physical heights are there?

- Orthometric heights
  - Height (H) along the plumbline
  - Needs the integrated gravity along the plumbline
  - Directly related to surveying
- Helmert orthometric heights
  - An approximation of orthometric heights based assumptions
- Normal heights
  - Approximates actual gravity with normal (ellipsoidal) gravity
- Dynamic Heights
  - Uses a fixed gravity value
  - Effectively scales the geopotential number into meters
  - Follows the flow of water but is not consistent with surveying

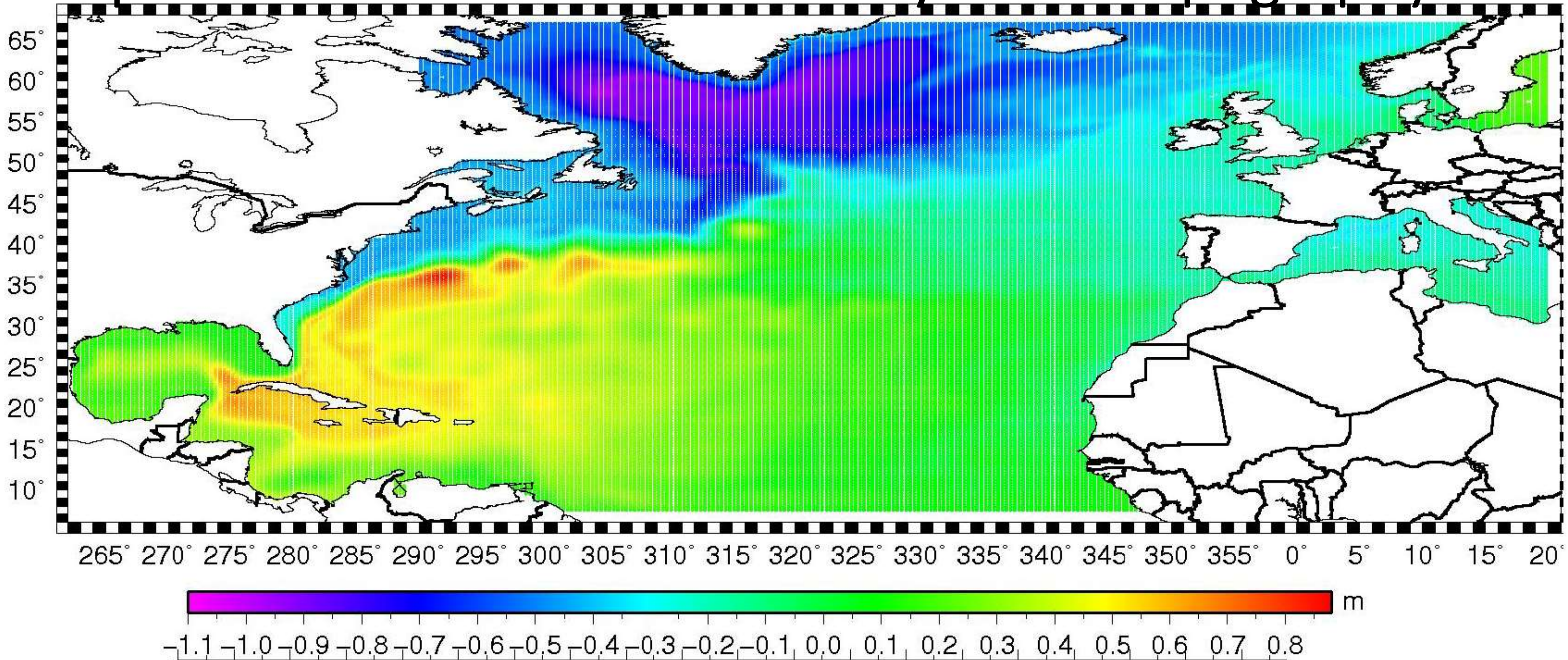


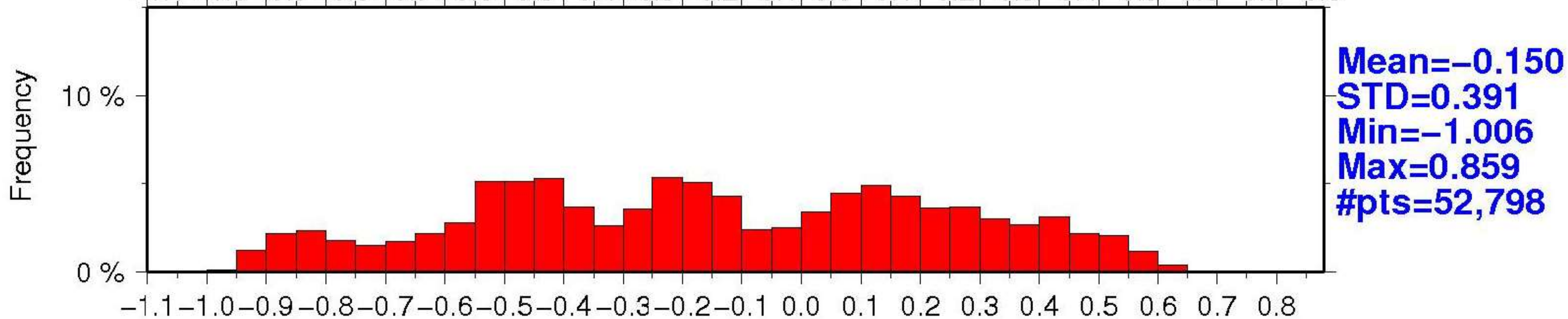

Note that surface location of station 1 is closer to the geoid than station 2.
A steep gradient of geops indicates higher gravity – less steep indicates lower gravity.
The geops being farther apart beneath station 2 to reflect lower local mass and gravity.
Hence, H1 should be less than H2 – even though both have the same geopotential.

# Geoid vs. Geoid Height

- Geoid
  - The equipotential surface of the Earth's gravity field which best fits, in the least squares sense, (global) mean sea level.
  - Can't see the surface or measure it directly.
  - Can be modeled from gravity data as they are mathematically related.
  - Note that the geoid is a vertical datum surface.
- Geoid Height
  - The ellipsoidal height from an ellipsoidal datum to a geoid.
  - Hence, geoid height models are directly tied to the geoid and ellipsoid that define them (i.e., geoid height models are not interchangeable).

# Thompson-Demirov Mean Ocean Dynamic Topography MODT



# Vertical Control Network NAVD 88

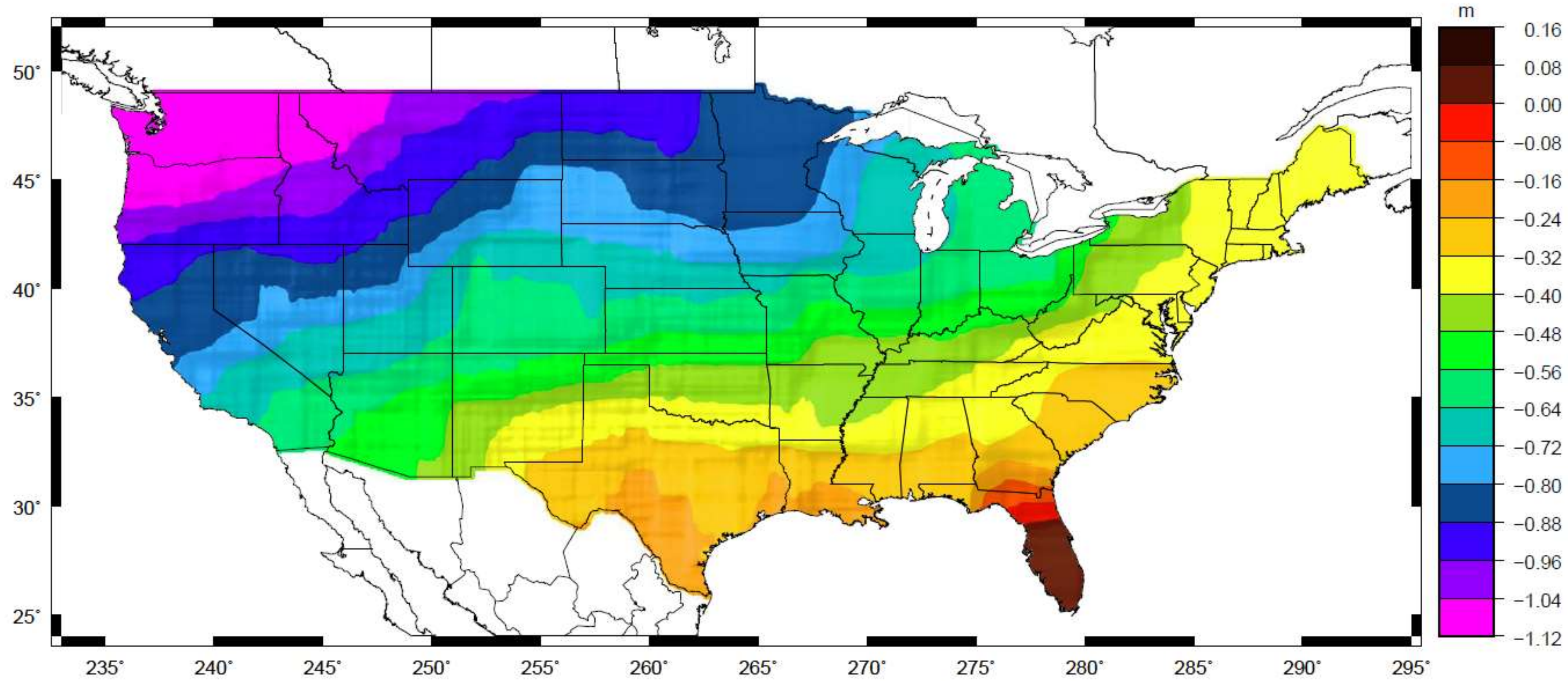

Figure 3. Vertical control used in 1988 adjustment.

**450,000 BM's over 1,001,500 km**

# North American Vertical Datum 1988 (NAVD 88)

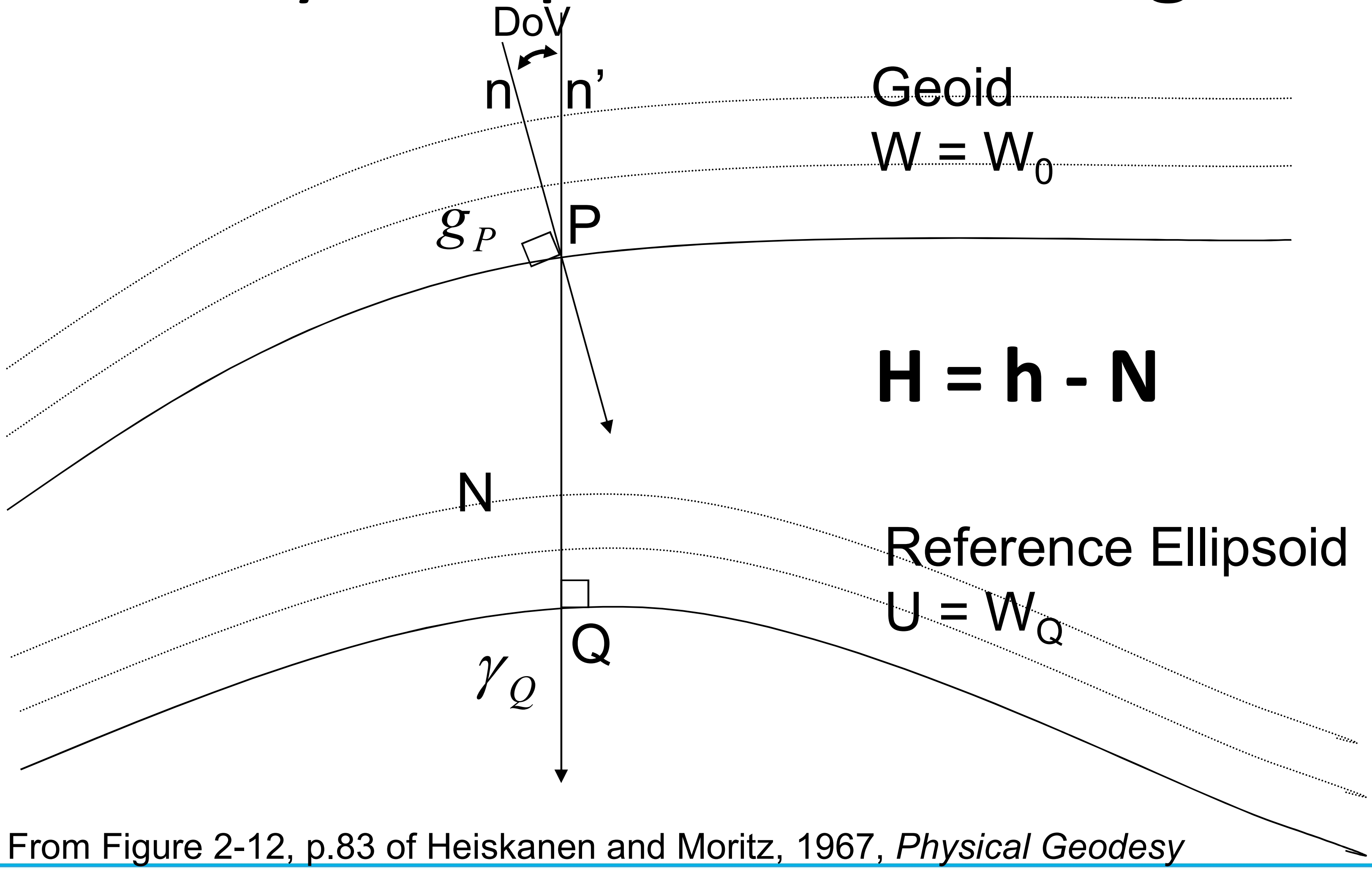
- Defined by one height (Father Point/Rimouski)
- Water-level transfers connect leveling across Great Lakes
- Adjustment performed in Geopotential Numbers
- Helmert Orthometric Heights:
  - $H = C / (g + 0.0424\ H_0)$
  - $C$ = geopotential number
  - $g$ = surface gravity measurement (mgals)
  - $H_0$ = approximate orthometric height (km)
- H = 0 level is nearly a level surface
- H = 0 level is biased and **tilted** relative to best available satellite-based geoid models

# Why isn't NAVD 88 good enough anymore?

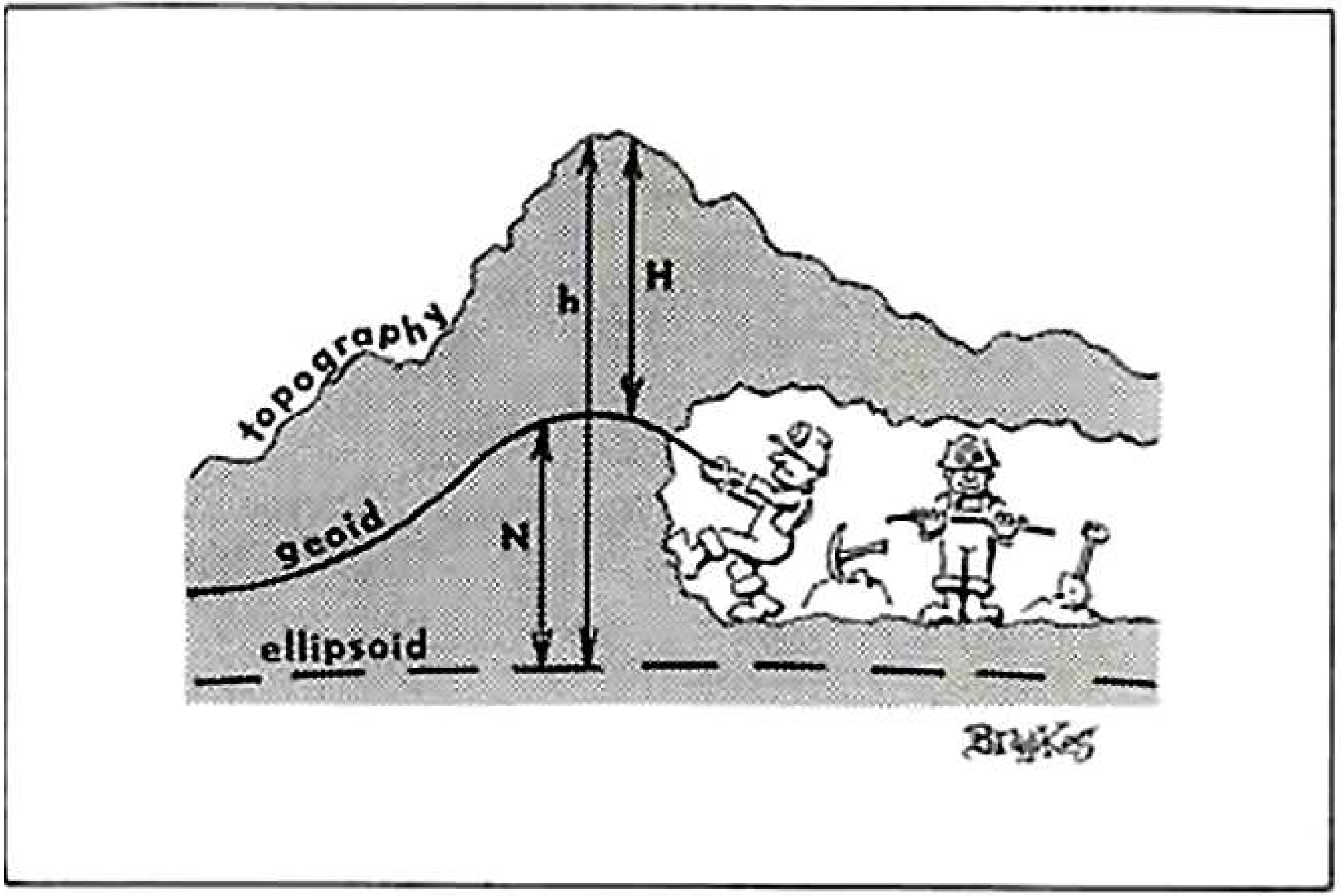



**Approximate level of error known to exist in the NAVD 88 zero elevation surface**

# Gravity, Geopotential & Heights




From Figure 2-12, p.83 of Heiskanen and Moritz, 1967, *Physical Geodesy*




In Search of the Geoid

Courtesy of Natural Resources Canada www.geod.nrcan.gc.ca/index_e/geodesy_e/geoid03_e.html

# Gravity, Geopotential & Heights

- Mass attracts other mass
- $M_E$ >>> any other mass
- Geopotential is 1/r
- Differential relationship
- Earth is more squashed
- So use *h* not *r*
- Then pick a datum ($W_0$)
- Physical heights relate to change in geopotential

- $g = Gm_1m_2/r^2$
- $g = GM\,(m)/r^2$
- $W = GM\,m/r$
- $g = \partial\,(W)/\,\partial r$
- geodetic not geocentric
- $g = \partial\,(W)/\,\partial h$
- $\int_0^P g\,\partial n = W_0 - W_P = C_P$
- $\Delta C_{b-a} = \sum_a^b h_i g_i$

# Gravity, Geopotential & Heights

- But the Earth is very big
- Need a good model: GRS80
- Work with residual values
  - Disturbing Potential
  - Gravity anomalies
  - Bruns Formula
  - Height relationships
  - Stokes' Formula

- Best to remove most signal
- It fits better than 99%
- Residual = actual – model

$T = W_P - V_P$

$\Delta g = g_P - \gamma_Q = \partial (T)/\partial h - 2T/h$

$N = T/g$

$h = H + N$

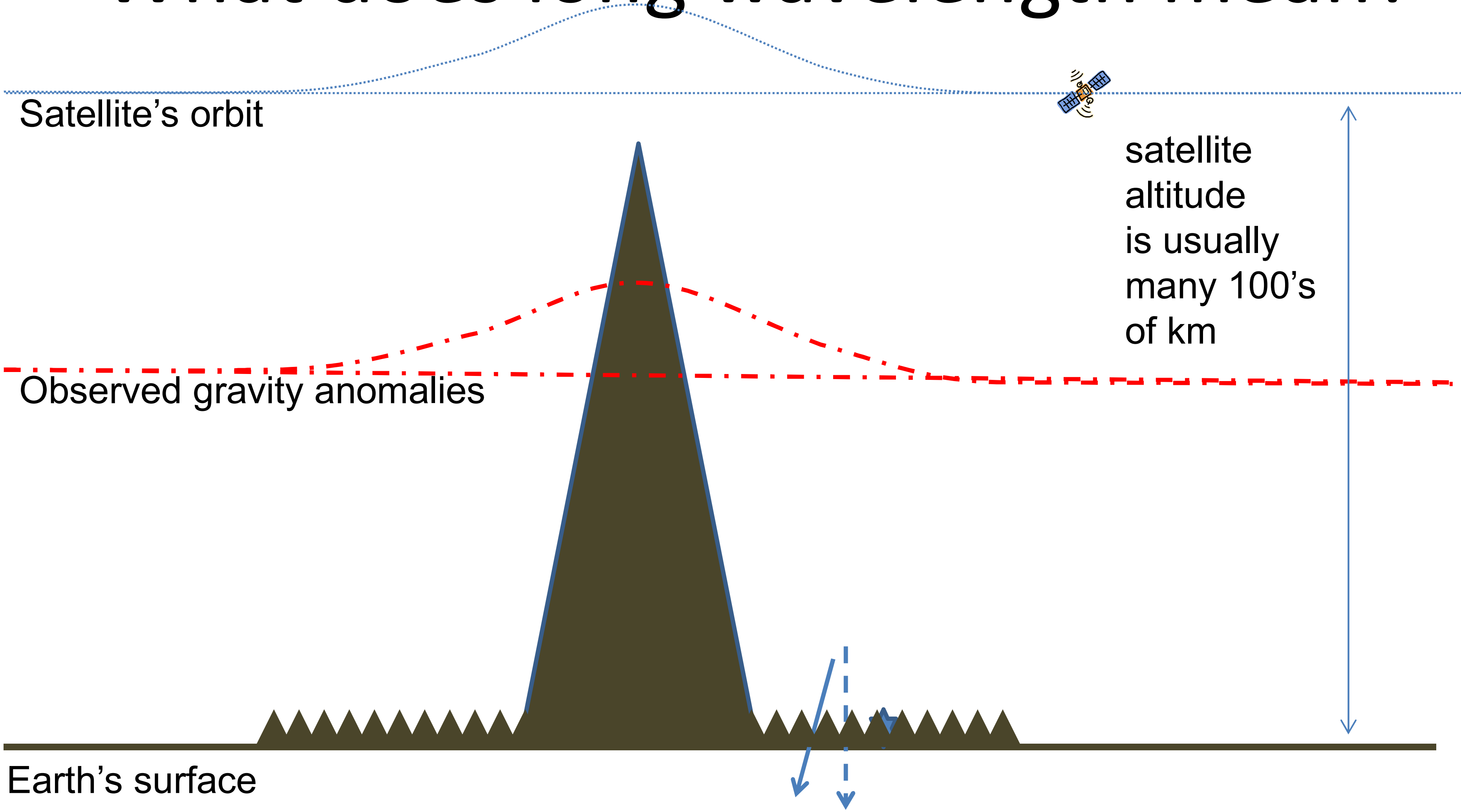
$$N = \frac{R}{4\pi G}\iint_{\sigma} \Delta g S(\psi) d\sigma$$

$$S(\psi) = \frac{1}{\sin\left(\psi/2\right)} - 6\sin\left(\frac{\psi}{2}\right) + 1 - 5\cos\psi - 3\cos\psi \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right)$$

# The Gravity Field Spectrum

- Concepts of scale
  - Long wavelengths: satellites
  - Short wavelengths: terrain
- Degree-Spectrum plot
  - Satellite gravity models
  - Aerogravity
  - Surface gravity observations
  - Terrain modeled gravity

# What does long wavelength mean?



# What does short wavelength mean?

**Dealing with residuals**

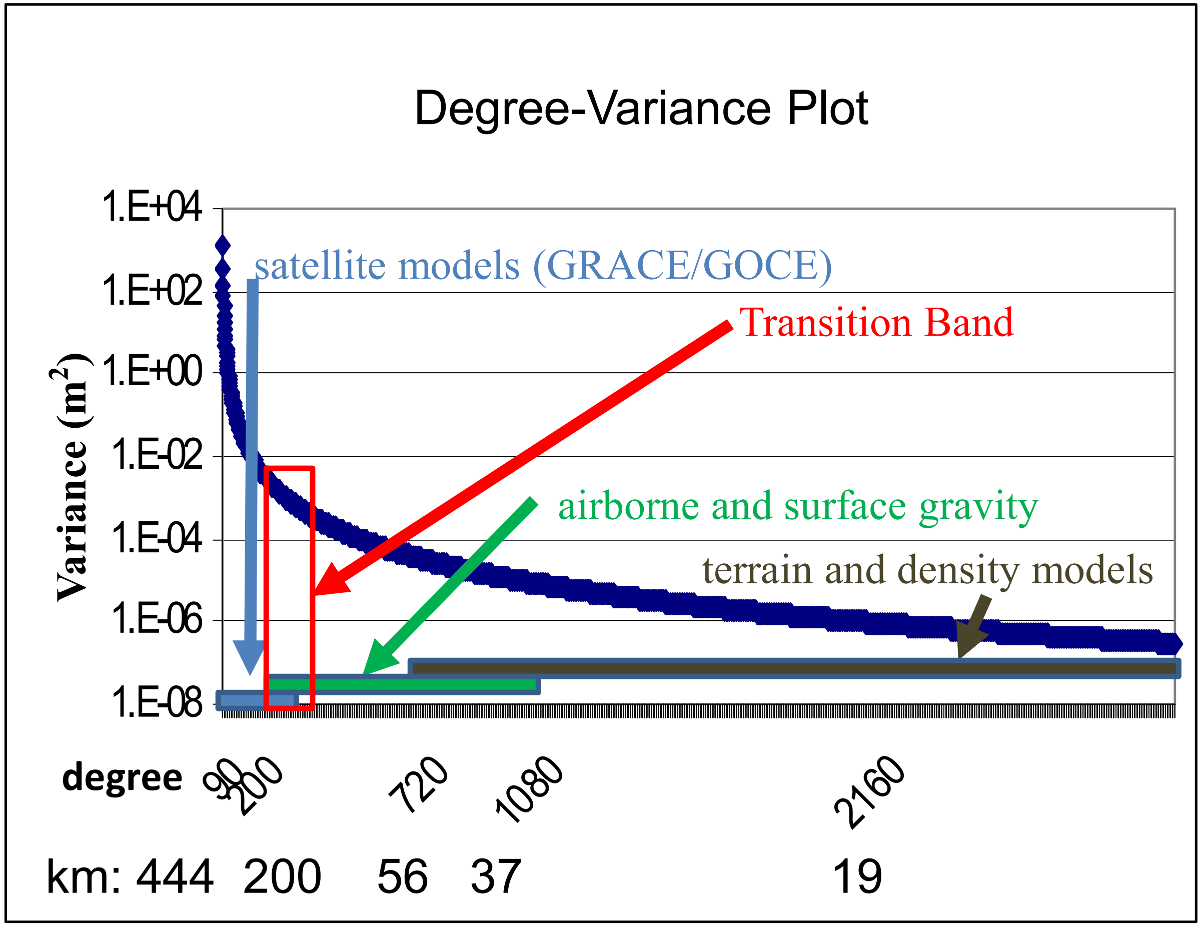
- gravity anomalies
- $\Delta g = g_P - \gamma_Q$
- Most signal removed ($\gamma$)
- Accounts for degree 2
- Leaves everything else
- Satellite models > 200 km
- Terrestrial gravity coverage not likely sufficient
- Must use DTM/DEM's

**Dealing with mountains**

- Residual Terrain Model
  - Models the terrain in SHM/EHM
  - Used in EGM2008 to 5'
  - Use SRTM 3" to get 3"-5'
  - 90 meter resolution
  - Signal < 3" affects Δg's
- Terrain Corrections
  - Used more with Stokes approach
  - Removes impact on gravity obs.
  - Then you make the geoid model

# The Gravity Field Spectrum

- SHM/EHM
- Inverse to scale
- d. 2 => Ellipsoid
  - 2 oscil. in circle
- Earth is 40,000 km round
- d. 360 is 1/360th of that or 111 km
- d. 2160 is 18 km



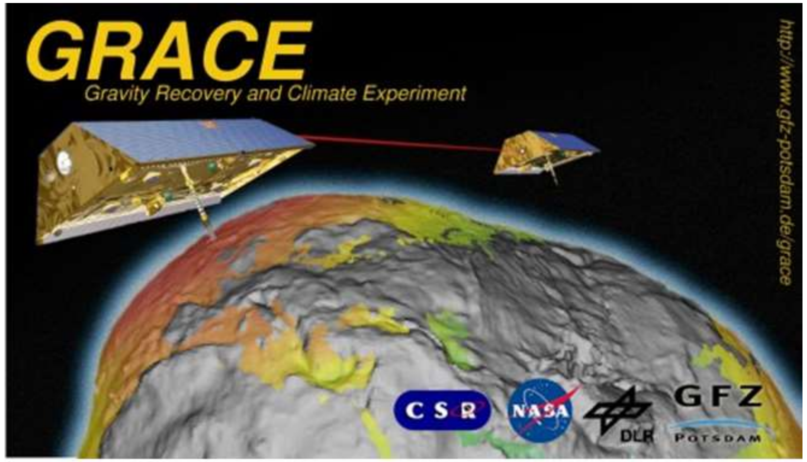
# Satellite Geodesy

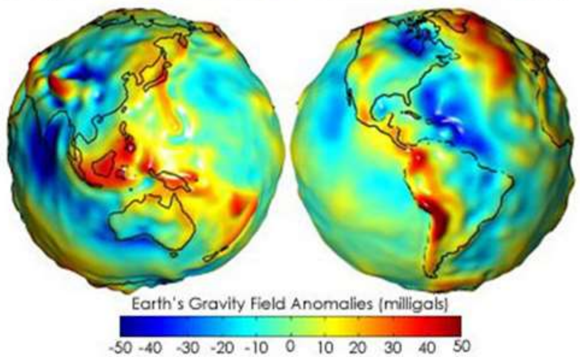
- Started with tracking satellites in orbit (60’s)
- First dedicated gravity mission was CHAMP
- GRACE was next (10 years on and still going)
- GOCE flying (20 months)
- Likely GRACE II/GFO & follow-on ESA missions

- Best approaches involve
  - GNSS receivers: hi-low
  - Low-low tracking (GRACE)
- Basic idea: orbital changes arise from gravity changes
- NOTE: gravity is observed in orbit <u>not</u> on the ground
- Also, orbital height is a function of sensitivity to scale (attenuation)

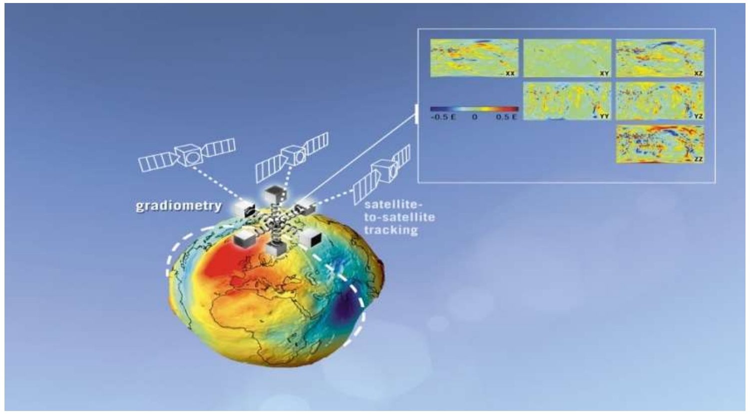
# Recent Satellite Gravity Missions

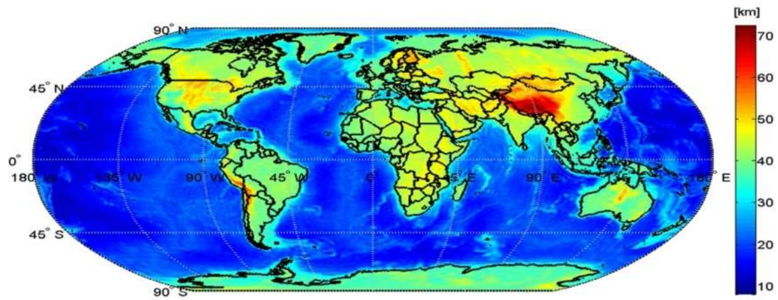
**GRACE**




**GOCE**



# International Height Reference System

- GGOS (IAG) effort led by Laura Sanchez
- Formally adopted by the IAG in 2015
- $W_0$ of geoid = best global MSL estimate = 62 636 853.4 $m^2s^{-2}$
- Based on satellite models for the long wavelengths
- Enhanced locally for additional gravity field information

"Definition and Proposed Realization of the International Height Reference System (IHRS)" Ihde, J., Sánchez, L., Barzaghi, R. et al. Surv Geophys (2017) 38: 549. https://doi.org/10.1007/s10712-017-9409-3

# Summary

- Local MSL variations (MODT) complicate selection of a single tide gauge for vertical datum definition
- Adjustment of large networks of vertical control can propagate significant errors and be difficult to maintain
- A geoid height model in conjunction with GNSS observations offers cheap access to potentially accurate vertical control
- Development of such models comes from combination of gravity field data from multiple sources
- An IHRS has been established to help unify height systems around the world

# Thank you for your attention

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