Egyptian Geoid using Ultra High-Degree Tailored Geopotential Model

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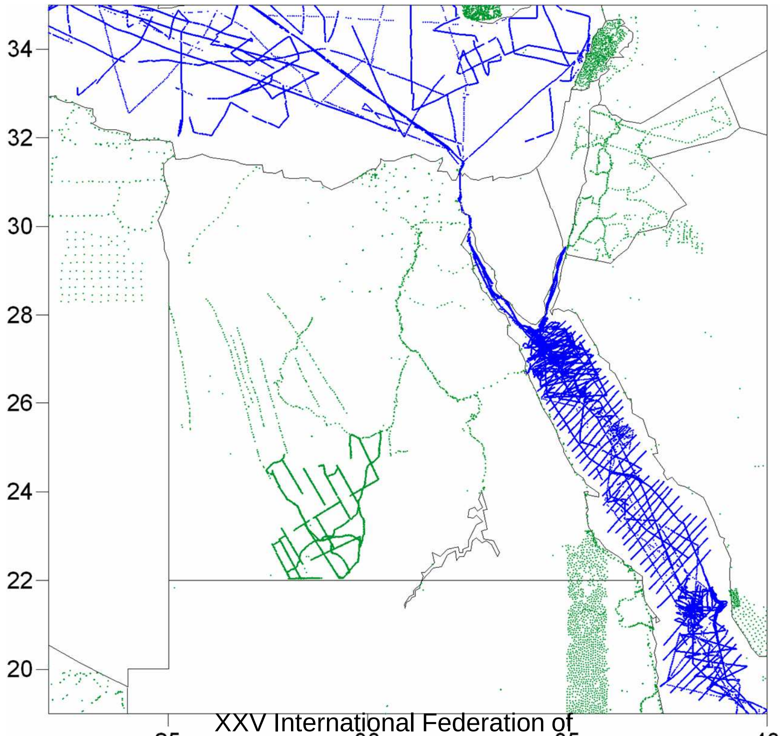
Data

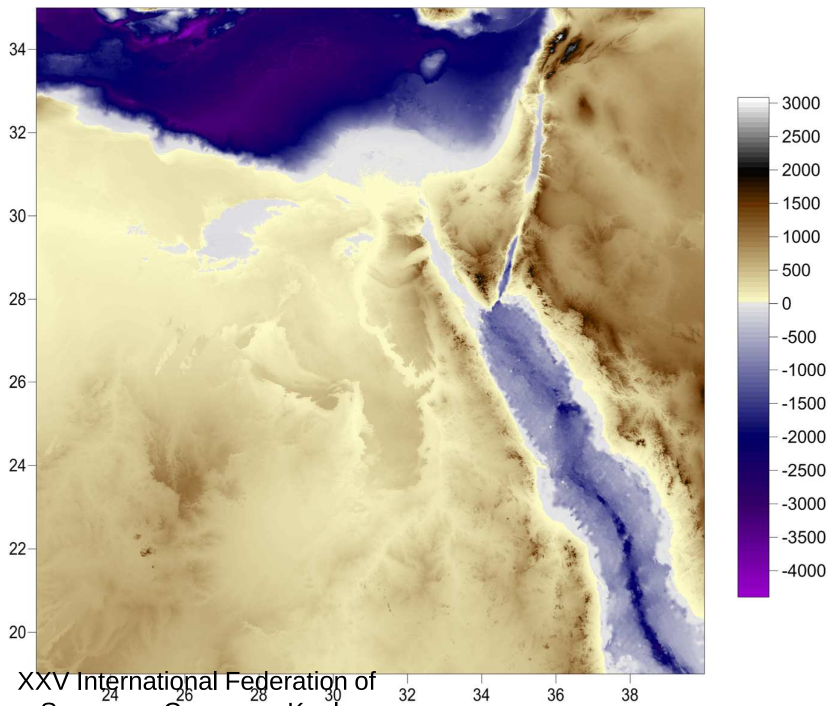
1. Gravity Data



Data

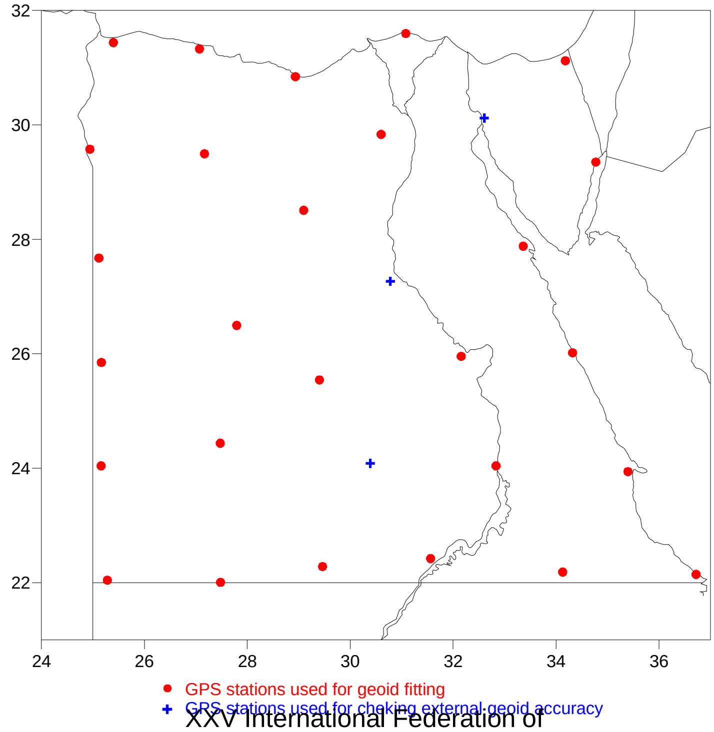
2. Digital Height Models (EGH13) (Abd-Elmotaal and Ashry, 2013)

- Coarse DHM resolution is 30" × 30"
- Fine DHM resolution is 3" × 3"



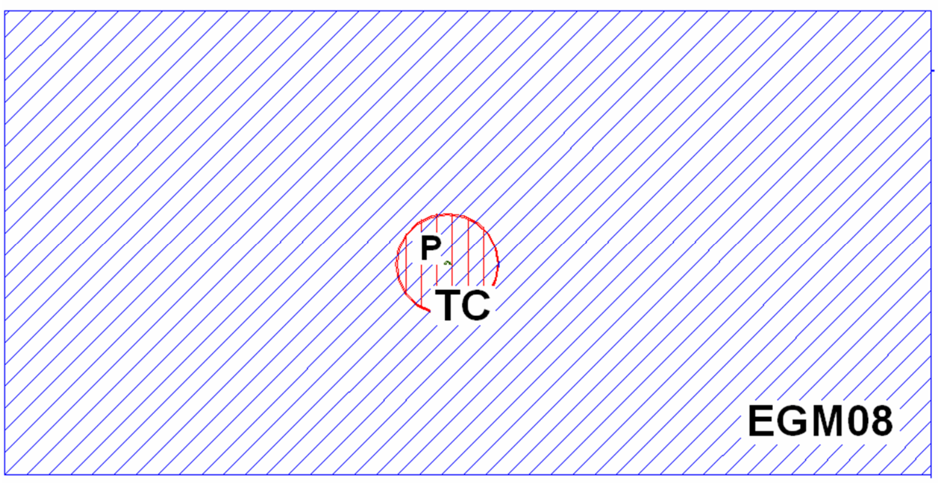
Data

3. GPS/Levelling Stations



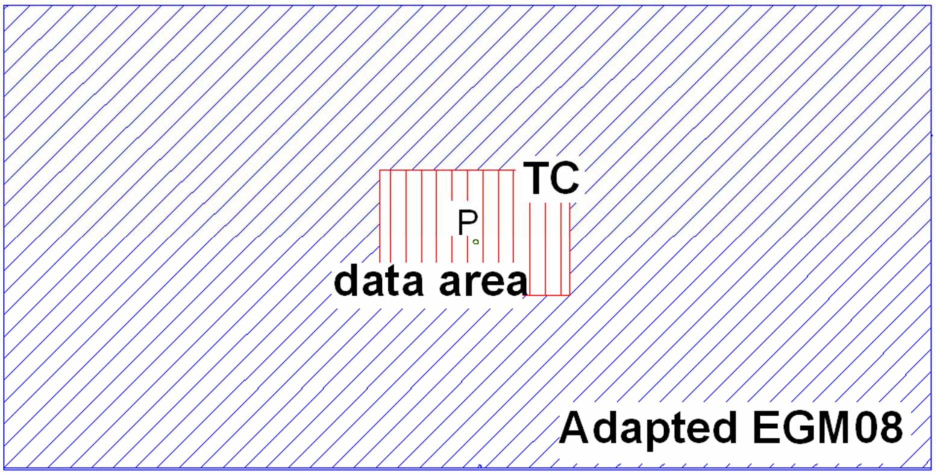
Remove/Restore Technique

1. Traditional Technique



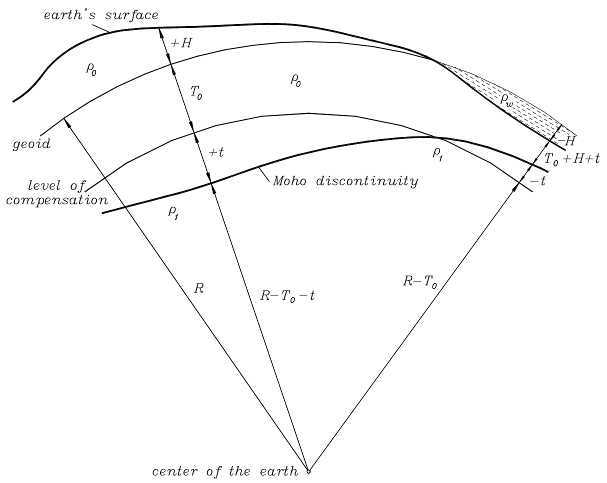
- Double consideration of the topographic-isostatic masses within the radius R_2 (double hatched area)

Remove/Restore Technique
2. Window Technique



- No double consideration of the topographic-isostatic masses (no double hatching)

Harmonic Analysis of the Topographic-Isostatic Potential



$$\begin{Bmatrix}\bar{C}_{nm}\\ \bar{S}_{nm}\end{Bmatrix} = \frac{R^3\Delta\phi\Delta\lambda}{M(2n+1)(n+3)}\sum_i^{\phi}\sum_j^{\lambda}\left\{\rho_{ij}\left[\left(1+\frac{H_{ij}}{R}\right)^{n+3}-1\right]\right.$$
$$\left. +\Delta\rho_{ij}\left[\left(1-\frac{D}{R}\right)^{n+3}\left(\left(1+\frac{kH}{R-D}\right)^{n+3}-1\right)\right]\right\}\begin{Bmatrix}\cos m\lambda_j\\ \sin m\lambda_j\end{Bmatrix}\bar{P}_{nm}(\cos\phi_i)\cos\phi_i$$

$$\rho_{ij} = \rho_0 \quad \text{for } H_{ij} > 0$$

$$\rho_{ij} = \rho_0 - \rho_w \quad \text{for } H_{ij} < 0$$

$$\Delta\rho_{ij} = \rho_0 - \rho_{ij} \quad \text{for all } H_{ij}$$

Methodology

$$\Delta g_{red} = \Delta g_F - \Delta g_{TC\,win} - \Delta g_{GM\ Adapt}$$

$$= \Delta g_F - \Delta g_{TC\,win} - \left(\Delta g_{GM} - \Delta g_h\right)$$

$$\Delta g_{GM} = \Delta g_{GM}\big|_{0\le n\le 360} + \Delta g_{GM}\big|_{361\le n\le 2160}$$

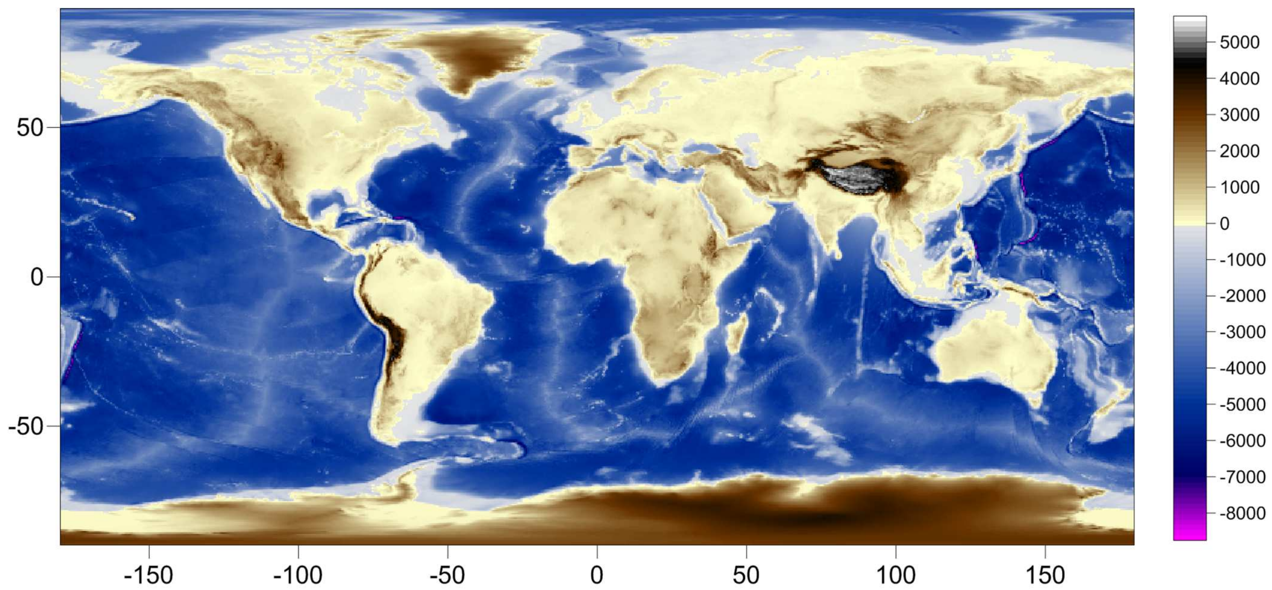
$$\Delta g_{red} = \Delta g_F - \Delta g_{TC\,win} - \Delta g_{GM}\big|_{0\le n\le 360} - \Delta g_{GM}\big|_{361\le n\le 2160} + \Delta g_h$$

$$\Delta g_{red} \equiv 0$$

$$\Delta g_{GM_T}\big|_{0\le n\le 360} = \Delta g_F - \Delta g_{TC\,win} - \Delta g_{GM}\big|_{361\le n\le 2160} + \Delta g_h$$

Preparing Gravity Anomalies for Tailored Geopotential Model Computation

1.1. SRTM 30' × 30' Global Digital Height Model



Min = -8756 m Max = 5719 m Mean = -1892 m St. dev. = 2638 m

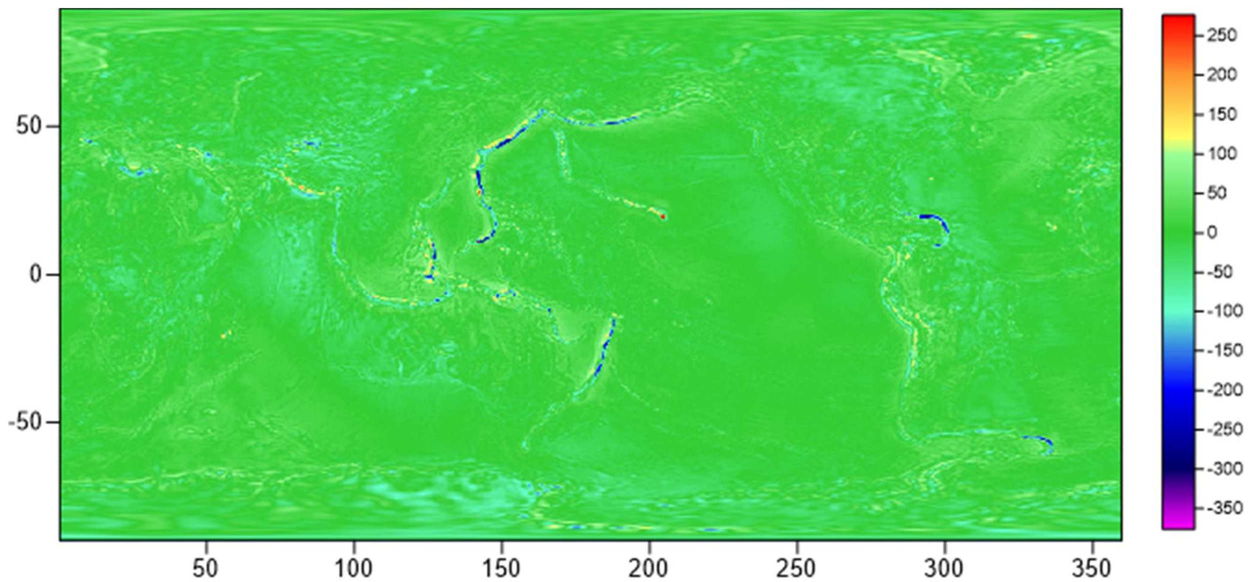
Preparing Gravity Anomalies for Tailored Geopotential Model Computation

1.2. EGM2008TI Topographic-Isostatic Geopotential Model

$$\begin{Bmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{Bmatrix}_{EGM\,2008TI} = \begin{Bmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{Bmatrix}_{EGM\,2008} - \begin{Bmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{Bmatrix}_{TI}$$

Preparing Gravity Anomalies for Tailored Geopotential Model Computation

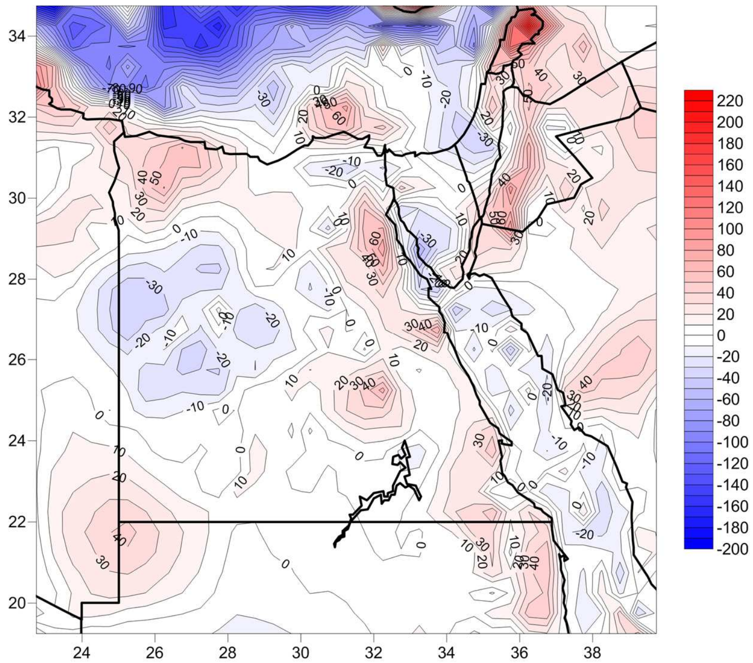
2. Global isostatic Anomalies (EGM2008TI)



Min = -337 mgal Max = 273 mgal Mean = -1 mgal St. dev. = 27 mgal

Preparing Gravity Anomalies for Tailored Geopotential Model Computation

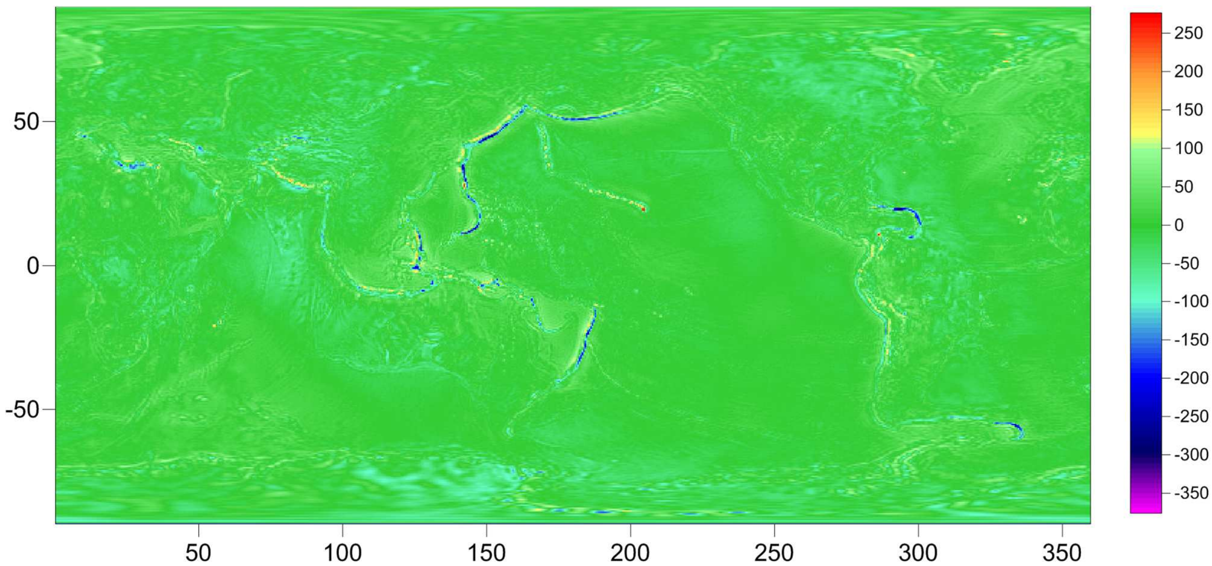
3. Local Isostatic (Free-air – window topo-iso) Gravity Anomalies (Egypt)



Min = -190 mgal Max = 202 mgal Mean = 0 mgal St. dev. = 37 mgal

Preparing Gravity Anomalies for Tailored Geopotential Model Computation

4. Merged Isostatic Gravity Anomalies (EGM2008TI + Egypt)



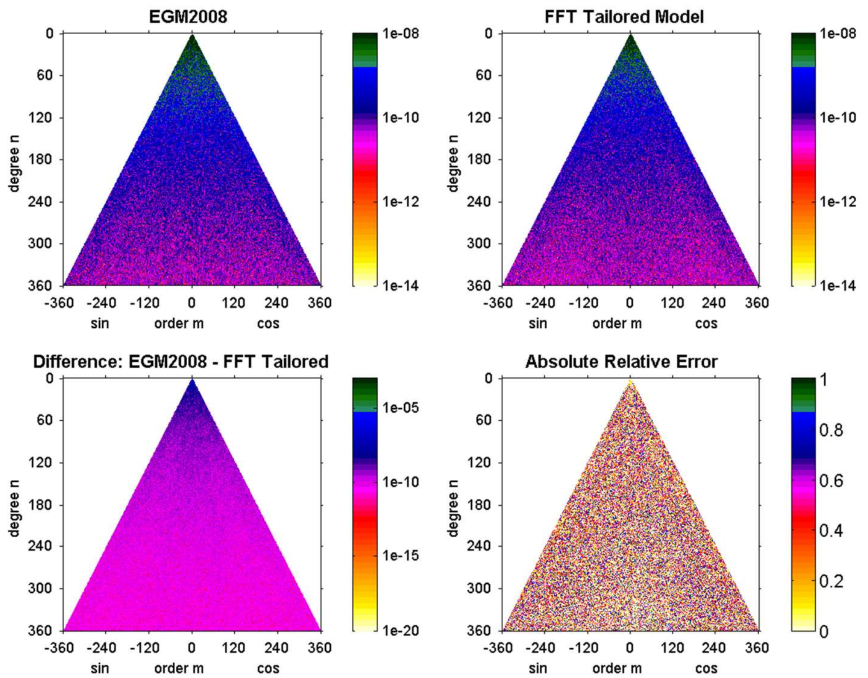
Min = -337 mgal Max = 273 mgal Mean = -1 mgal St. dev. = 27 mgal

Harmonic Analysis using FFT Technique

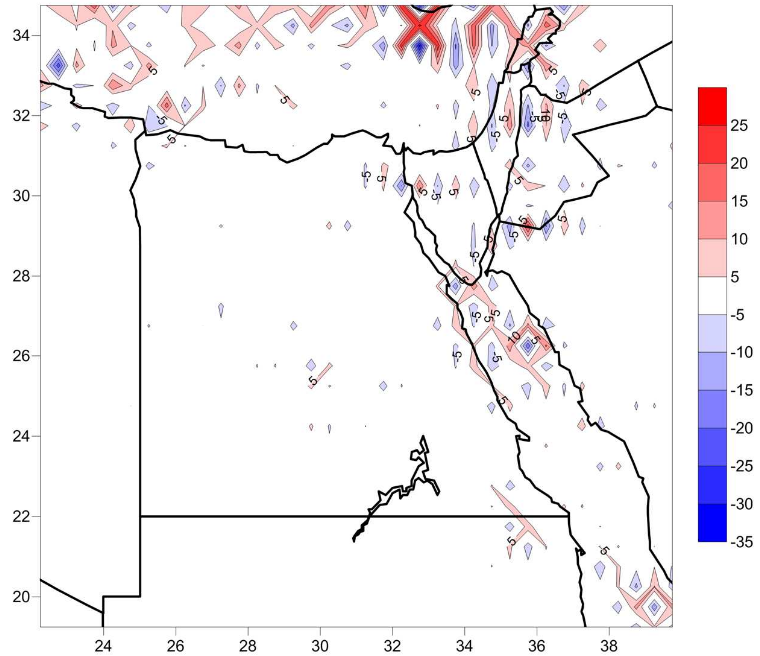
- Expanding $f(\theta,\lambda) \rightarrow$ $$f(\theta,\lambda)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\overline{F}_{nm}\,\overline{Y}_{nm}(\theta,\lambda)$$
- Orthogonality $\rightarrow$ $$\overline{F}_{nm}=\frac{1}{4\pi}\iint_{\sigma} f(\theta,\lambda)\,\overline{Y}_{nm}(\theta,\lambda)\,d\sigma$$
- HRCOFITR Program (Abd-Elmotaal, 2004 b) has been used
- Iteration to obtain the best coefficients accuracy and minimum residual field

Tailored Reference Geopotential Model for Egypt (EGTGM2014)

(lower harmonics only till n=360; model is available till n=2160)



Residual Field (Data – Model)

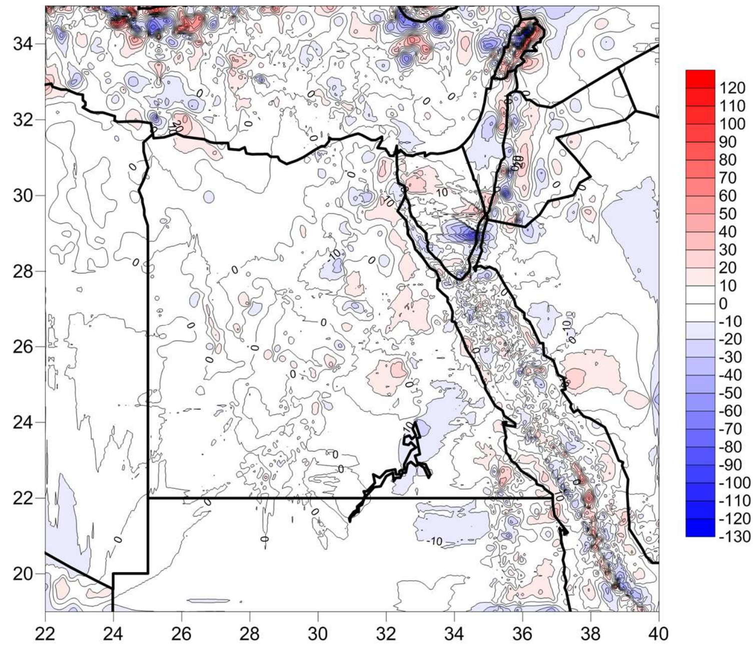


Min = -32.4 mgal Max = 27.6 mgal Mean = 0 mgal St. dev. = 5.1 mgal

Gravity Reduction (102418 points)

Reduced gravity	Minimum	Maximum	Average	St. dev.	Variance
Δg_F	-210.60	314.99	-27.58	50.65	2565.1
$\Delta g_{Airy\ window}$ (EGM2008)	-99.15	122.49	-0.26	20.46	418.6
$\Delta g_{Airy\ window}$ (EGTGM2014)	-134.30	112.69	-0.45	16.45	270.5

Airy window isostatic anomalies for Egypt using the EGTGM2014 tailored geopotential model



Min = -134.3 mgal Max = 112.7 mgal Mean = -0.5 mgal St. dev. = 16.5 mgal

Stokes' Integral with 1D-FFT Technique

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma} \Delta g\, S(\psi)\, d\sigma$$

$$N(\phi_P,\lambda_P) = \frac{R\,\Delta\phi\,\Delta\lambda}{4\pi\gamma}\sum_{Q}(\Delta g_Q \cos\phi_Q)\, S(\psi_{PQ})$$

$$N_{\phi_P}(\lambda) = \frac{R\,\Delta\phi\,\Delta\lambda}{4\pi\gamma}\sum_{\phi_Q}\cos\phi_Q \sum_{\lambda_Q}\Delta g_Q\, S(\psi_{PQ})$$

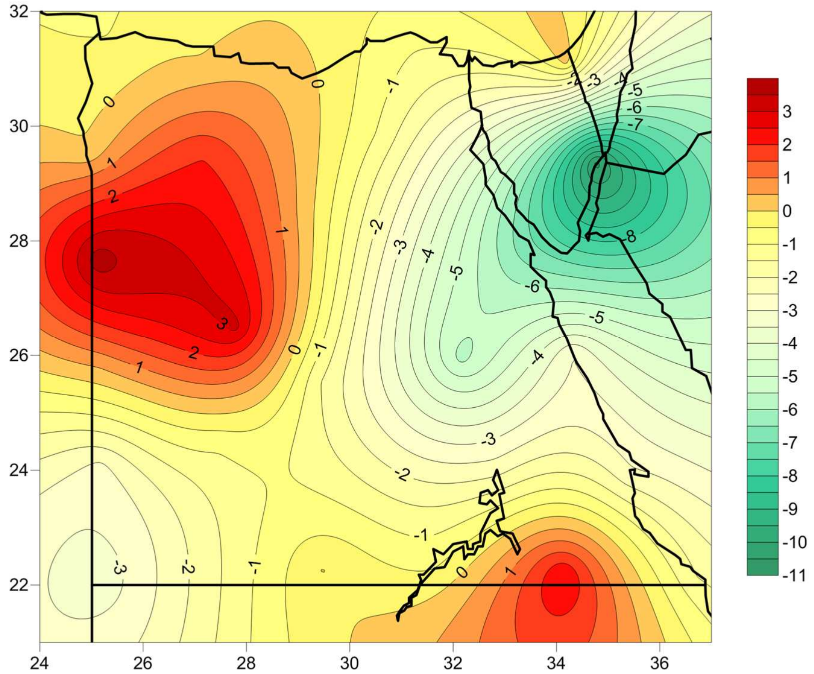
$$S(\psi_{PQ}) = S_{\phi_P\phi_Q}(\lambda_P-\lambda_Q) = S_{\phi}(\lambda_P-\lambda_Q)$$

$$N_{\phi_P}(\lambda) = \frac{R\,\Delta\phi\,\Delta\lambda}{4\pi\gamma}\sum_{\phi_Q}\cos\phi_Q \sum_{\lambda_Q}\Delta g_Q\, S_{\phi}(\lambda_P-\lambda_Q)$$

$$N_{\phi_P}(\lambda) = \frac{R\,\Delta\phi\,\Delta\lambda}{4\pi\gamma}\,\mathcal{F}_1^{-1}\left\{\sum_{\phi_Q}\mathcal{F}_1\left(\Delta g_Q \cos\phi_Q\right)\mathcal{F}_1\left[S_{\phi}\left(\lambda_P-\lambda_Q\right)\right]\right\}$$

Absolute Geoid Difference to GPS/levelling

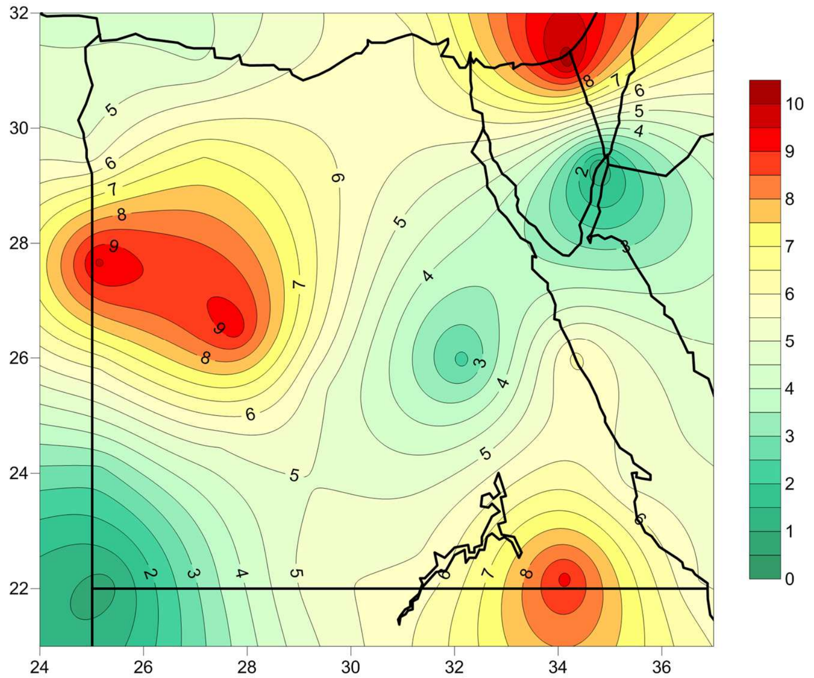
$$N_{GPS} - N_{Airy\,win\,EGM\,2008}$$



Min = -10.65 m Max = 3.78 m Mean = -1.20 m St. dev. = 3.00 m

Absolute Geoid Difference to GPS/levelling

$$N_{GPS} - N_{Airy\,win\,EGTGM\,2014}$$



Min =0.80 m Max = 10.24 m Mean = 5.44 m St. dev. = 2.33 m

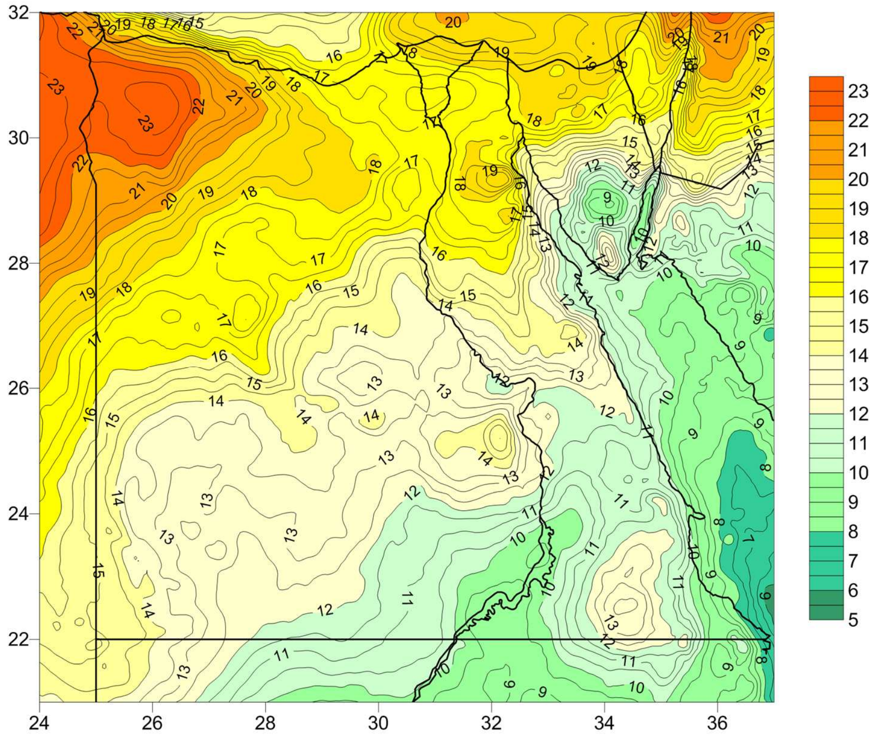
Statistics of the Remaining Differences at the 27 GPS Stations used for the Geoid Fitting after Removing a Kriging Trend Function (Internal Precision)

Geoid type	Minimum [cm]	Maximum [cm]	Average [cm]	Standard deviation [cm]
$N_{Airy\,win\,EGM\,2008}$	-9.2	9.8	0.2	3.4
$N_{Airy\,win\,EGTGM\,2014}$	-7.6	9.8	0.2	3.3

Statistics of the Remaining Differences after Removing a Kriging Trend Function at the 3 GPS Stations which were not used for the Geoid Fitting (External Accuracy)

Geoid type	Minimum [m]	Maximum [m]	Average [m]	Standard deviation [m]
$N_{Airy\,win\,EGM\,2008}$	-1.40	2.46	0.29	1.97
$N_{Airy\,win\,EGTGM\,2014}$	-0.88	2.13	0.30	1.60

Gravimetric Geoid for Egypt using EGTGM2014



Min = 5.50 m Max = 23.69 m Mean = 14.41 m St. dev. = 3.78 m

Summary and Conclusions

- An ultra high-degree tailored reference geopotential model for Egypt, complete to degree and order 2160, has been developed.
- The tailored geopotential model created in this investigation gives better residual gravity anomalies (unbiased and have much less variance). The variance has dropped by about 35%.
- Gravimetric geoids for Egypt have been computed in this investigation using both the EGM2088 and the EGTGM2014 tailored geopotential models in the framework of the window remove-restore technique using 1D-FFT technique.
- The computed geoids have been fitted to the GPS-derived geoid by removing a trend surface.
- A kriging trend function has been computed using only 27 GPS stations among the available 30 GPS stations in Egypt.
- The internal precision of the fitted geoids is very good (about 3 cm) and it is nearly equal for both geoids.
- Using the EGTGM2014 tailored geopotential model improves the external geoid accuracy by about 20%, and the range of the remaining differences has dropped by about 22%.

Thank You!