The Common Adjustment of GPS and Photogrammetric Measurements

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Key words: GPS, photogrammetry, multi-sensor integration

SUMMARY

GPS controlled aerial photogrammetry is, in its current guise, a mature technology that has found near universal acceptance in the mapping community. The current integration strategy is to first process the GPS data using a stand-alone processor, and then to use the resulting positions as parameter observations in a photogrammetric bundle adjustment. This implementation has obvious benefits in its simplicity; however, a more fundamental fusion of the GPS and photogrammetric data streams is possible. In this paper, investigations are made into a single combined adjustment that natively uses both photogrammetric image measurements and raw GPS code and carrier-phase observations. The anticipated advantages of this new integration technique include improved reliability and the ability to make use of GPS data when less than four satellites are available. The technique also streamlines processing as only a single software package need be used. Background and details are provided on existing integration techniques, on the revised collinearity equations that facilitate the inclusion of GPS observations and on the undifferenced and double-differenced code and carrier phase range observations used in the combined adjustment. Design details of the hierarchical adjustment software created to perform the combined adjustment are provided, with specific attention given to the GPS adjustment component. Through tests, the combined adjustment is compared against the conventional integration strategy in a variety of configurations of input data. The tests are not conclusive, but appear to indicate that the new technique is no more accurate than the old technique.

The Common Adjustment of GPS and Photogrammetric Measurements

Cameron ELLUM and Naser EL-SHEIMY, Canada

1. INTRODUCTION

Kinematic GPS controlled aerial photogrammetry has become an omnipresent technology in both the scientific and commercial mapping communities. Virtually all airborne mapping systems now integrate a GPS receiver with their camera. This integration is done at the hardware level, as the GPS receiver and camera must communicate, either for the GPS to trigger the camera or for the camera to record the exposure time. Unfortunately, on the software side, the integration of GPS and photogrammetry is not as close. Typically, the GPS data is included in the photogrammetric bundle adjustment only as processed positions (see, for example, Ackermann, 1992; Mikhail et al., 2001). Beyond this simple sharing, the GPS and photogrammetric processing engines operate entirely in isolation from each other. This implementation has obvious benefits in its simplicity and ease of implementation; however, a more fundamental fusion of the GPS data into the bundle adjustment may provide improvements in both accuracy and reliability.

This paper outlines a tighter coupling of the GPS and photogrammetric processing engines where the GPS code range and carrier phase measurements are directly included in the same adjustment as the photogrammetric observations. The goal of this integration is to improve the accuracy and reliability when compared to the naïve inclusion of GPS positions.

2. BACKGROUND

The theoretical foundations of GPS assisted aerial photogrammetry date back nearly 30 years, and the practice itself has been in widespread operation for well over a decade. The utility of GPS for assisting in aerial photogrammetry, together with the basic concept that is still followed near universally today, were first envisioned in the mid-seventies (Brown, 1976). This occurred even as GPS itself was still in its early planning stages. Naturally, the first tests of the technique had to wait until enough satellites had been placed in orbit, but by the mid eighties tests were being done using the partial satellite constellation. By the mid-nineties, the technique had made the move from academia to industry, and conferences from the period are replete with papers from commercial mapping companies describing their practical experiences with GPS assisted aerial photogrammetry. Effectively, by the end of the 1990s, the technique had become ubiquitous throughout both the academic and commercial mapping communities.

2.1. Including GPS observations via position observations

The technique near universally applied for combining GPS and photogrammetric data is the use of GPS position observations in photogrammetric bundle (block) adjustments. In this method, the raw GPS measurements are first processed using an external kinematic GPS processing program that provides position and covariance estimates. These positions are then included in bundle adjustments using simple parameter observation equations. Nominally, these equations resemble

$$\boldsymbol{r}_{GPS}^{M}(t) = \boldsymbol{r}_{c}^{M}(t) + \boldsymbol{R}_{c}^{M}(t)\boldsymbol{r}_{GPS}^{c}, \qquad (1)$$

where $\boldsymbol{r}_{GPS}^{M}(t)$ is the GPS position observation that is related to the camera perspective centre $\boldsymbol{r}_{c}^{M}(t)$ through the camera-GPS antenna lever-arm $\boldsymbol{r}_{GPS}^{c}$. $\boldsymbol{R}_{c}^{M}(t)$ is the rotation matrix that aligns the reference frame of the camera with that of mapping space. In practice, the GPS observations don't correspond with the actual exposure times, and so the exposure positions must be interpolated from adjacent GPS positions. Also, the position accuracy estimates from the GPS processor are frequently optimistic, and so they should be scaled to make their weights consistent with the weights of other observations in the adjustment.

Equation (1) is frequently augmented to include bias and time-dependent linear drift terms,

$$\boldsymbol{r}_{GPS}^{M}(t) = \boldsymbol{r}_{c}^{M}(t) + \boldsymbol{R}_{c}^{M}(t)\boldsymbol{r}_{GPS}^{c} + \left(\boldsymbol{b}_{GPS}^{M} + \boldsymbol{d}_{GPS}^{M}(t - t_0)\right). \qquad (2)$$

These two terms, denoted as $\boldsymbol{b}_{GPS}^{M}$ and $\boldsymbol{d}_{GPS}^{M}$, respectively, are primarily intended to account for incorrect ambiguity resolution in the external GPS processor, as it is assumed this and other GPS errors manifest themselves linearly in the GPS positions. Each strip of imagery gets its own set of these parameters. If ground control is also used in the adjustment, then the shift and drift terms can also account for inconsistencies between the datum and the GPS positions. Indeed, with modern receivers and processing software it is reasonable to conclude that the shift and drift terms are more likely compensating for datum shifts and other errors than they are modelling incorrectly resolve ambiguities.

As evidenced by its widespread acceptance, including GPS data in the adjustment via position observations has a number of advantages; however, it is not without its problems. On the plus side, it is both conceptually simple and easy to implement. The photogrammetric adjustment software requires only minimal changes and no changes at all are required to the GPS processor. On the minus side, because the processing of the GPS data is done in complete isolation from the photogrammetric processor, it doesn't benefit from the photogrammetric information. Moreover, combating GPS errors using the shift and drift approach presupposes that GPS errors manifest themselves as linear errors in the positions. In reality, this is not always the case (Jacobsen & Schmitz, 1996). Also, introducing the shift and drift parameters in the adjustment necessitates cross strips being flown, measured, and adjusted, otherwise the parameters are not determinable. Finally, using position observations has several practical drawbacks, including the requirement for operators to have expertise with two software packages and the difficulties can arise with transferring results from the GPS processor to the bundle adjustment.

2.2. Including GPS observations

To the author's knowledge, the only other investigation into a different technique for integrating GPS and photogrammetry was initiated at University of Hanover and Geo++ GmbH in the mid-nineties. In their ingenious approach, outlined in Jacobsen and Schmitz (1996) and Kruck et al. (1996), constant satellite-to-exposure station range corrections are estimated within the bundle adjustment for each GPS satellite whose ambiguity was not reliably fixed in the kinematic GPS processor. The development of this technique begins with the linearised GPS range observation equations, where the additional range corrections $\boldsymbol{\Delta l}$ are explicitly separated from the range measurements ***l***,

$$\boldsymbol{l} + \boldsymbol{\Delta l} = \boldsymbol{A}\boldsymbol{x}. \tag{3}$$

In Equation (3), ***x*** is the vector of GPS co-ordinates and ***A*** is the GPS design matrix (i.e., partial derivatives of the geometric range with respect to the co-ordinate components). Solving this equation by least squares yields

$$\boldsymbol{x} = \left(\boldsymbol{A}^T \boldsymbol{P}\boldsymbol{A}\right)^{-1} \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{l} + \left(\boldsymbol{A}^T \boldsymbol{P}\boldsymbol{A}\right)^{-1} \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{\Delta l}. \tag{4}$$

This equation has two terms: the first is the GPS co-ordinate vector that would be solved for in the absence of the ***Δl*** range corrections, and the second is a vector of co-ordinate corrections that results because of these range corrections. This second term is introduced into the bundle adjustment's GPS position observation equation, effectively replacing the shift and drift terms from the conventional approach,

$$\boldsymbol{r}_{GPS}^{M}(t) = \boldsymbol{r}_{c}^{M}(t) + \boldsymbol{R}_{c}^{M}(t)\boldsymbol{r}_{GPS}^{c} + \left(\boldsymbol{A}^T \boldsymbol{P}\boldsymbol{A}\right)^{-1} \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{\Delta l}. \tag{5}$$

The ***Δl*** range corrections are then are added to the bundle adjustment as unknown parameters.

This integration technique has several advantages over the traditional position observation GPS/photogrammetry integration strategy, yet it is not quite the "rigorous" integration claimed. Improvements over the traditional approach include:

- the actual GPS errors are better considered
- the number of unknowns is (in general) reduced
- no cross-strips are required
- GPS errors can better be separated from datum and interior orientation parameters

In spite of these advantages it is, however, important to note that the only GPS information introduced into the bundle adjustment is geometric in nature. The actual GPS ranges themselves are not used and so the integration is effectively still done in object space. In other words, because the actual GPS measurements are not directly used in the adjustment, the integration is still incomplete. Also, the sharing between the GPS and photogrammetric processors is, like in the conventional approach, only in one direction. In fairness, the creators of the technique do note that "re-substitution of the [range correction] terms [into the GPS processor] is feasible"; however, they conclude that "it is not of much interest as the GPS processing techniques improve" (Jacobsen & Schmitz, 1996).

2.3. Combined Adjustment of GPS and photogrammetric measurements

The integration of GPS and photogrammetry is only truly complete when it is done at the measurement level, and a combined adjustment is the easiest way to accomplish this. In a combined adjustment all the measurements are input into a single least squares adjustment. This is, admittedly, conceptually simple, but, to the author's knowledge, has not been mentioned before in either the GPS or photogrammetric literature.

The combined adjustment integration strategy should provide a number of benefits. Perhaps the most anticipated of these is improved reliability; in particular, an improved ability to detect GPS errors. Another important practical benefit is faster, simpler, and more streamlined processing, as familiarity is only required with a single software package. The combined adjustment will also enable GPS data to be used when data from less than four satellites is available, which is not the case in current integration strategies. While not particularly relevant for airborne mapping, this may have applicability in terrestrial mapping systems. Obviously, another key benefit hoped for was an increase in mapping accuracy. However, initial results indicate that this may not be the case. Further details are provided in Section 5.

The combined adjustment has, of course, several disadvantages. For example, it is not possible to make use of a kinematic model as is done in a GPS Kalman filter. Also, implementing the combined adjustment requires significant effort. Finally, there are important and as yet unknown issues with regards to the correct relative weighting of the different observation types.

3. THEORY

A combined adjustment of photogrammetric and GPS measurements has relatively minor theoretical novelty. This is because the theory behind the individual adjustment of both photogrammetric and GPS measurements is well-established.

3.1. Modification of the Collinearity Equations

The collinearity equations are the basis of analytical photogrammetry. Conventionally, they describe the relationship between an object space point, an image measurement of that point, and the perspective centre of a camera. In a combined adjustment, however, it is convenient to recast them so that they are explicitly functions of GPS co-ordinates associated with the exposure stations. This is advantageous because the GPS observation equations are also functions of the GPS co-ordinates, and making the collinearity equations the same facilitates the inclusion of the GPS equations.

Derivation of the modified collinearity equations used in the combined adjustment starts with the forward conformal transformation that relates the GPS positions with the image co-ordinates,

$$\boldsymbol{r}_P^M = \boldsymbol{r}(t)_{GPS}^M - \boldsymbol{R}_c^M(t)\left(\boldsymbol{r}_{GPS}^c - \mu_p^P \boldsymbol{r}_p^c\right), \tag{6}$$

By rearranging Equation (2), the reverse transformation is found to be

$$\boldsymbol{r}_p^c = {}^{1}\!\big/_{\mu_p^P}\left[\boldsymbol{R}_M^c\left(\boldsymbol{r}_P^M - \boldsymbol{r}(t)_{GPS}^M\right) + \boldsymbol{r}_{GPS}^c\right]. \tag{7}$$

Elimination of the third equation yields the modified collinearity equations that are explicitly functions of the GPS co-ordinates,

$$\begin{aligned} x_p &= \frac{r_{11}(X_p - X_{GPS}) + r_{12}(Y_p - Y_{GPS}) + r_{13}(Z_p - Z_{GPS}) + x_{GPS}}{r_{31}(X_p - X_{GPS}) + r_{32}(Y_p - Y_{GPS}) + r_{33}(Z_p - Z_{GPS}) + z_{GPS}} \\ x_p &= \frac{r_{21}(X_p - X_{GPS}) + r_{22}(Y_p - Y_{GPS}) + r_{23}(Z_p - Z_{GPS}) + y_{GPS}}{r_{31}(X_p - X_{GPS}) + r_{32}(Y_p - Y_{GPS}) + r_{33}(Z_p - Z_{GPS}) + z_{GPS}} \end{aligned}, \tag{8}$$

These modified collinearity equations were first identified in Ellum (2001).

As noted above, the chief advantage of the modified collinearity equations is that they ease the later inclusion of the GPS observation equations; however, there are a number of other advantages as well. For instance, because the GPS positions are one of the quantities being adjusted, the GPS positions can be directly used as parameter observations without including the GPS lever arm as in Equation 1. Adjusting the GPS positions directly also means that they are one of the quantities output by the adjustment. This allows for easy comparison with the input positions, which in turn simplifies the analysis of the results. Finally, if the modified collinearity equations are used in multiple-camera systems (for example, terrestrial van systems), then all the cameras can be referenced to a common position.

3.2. GPS Observation equations

Inclusion of the GPS code and carrier phase measurements in the adjustment is done using conventional GPS observation equations. It is possible to include any type of GPS observation, but currently only undifferenced code range measurements and double-differenced code and carrier phase measurements have been examined. For undifferenced code range measurements, the observation equation is

$$\mathrm{p} = |\, \mathbf{r}_{GPS/SV} \,| + c\, \Delta t_{rx}, \tag{9}$$

with p the code range measurement, $\mathbf{r}_{GPS/SV}$ the vector of antenna-to-satellite co-ordinate differences, c the speed of light, and Δt_{rx} the receiver clock bias. This last term is added to the adjustment as an unknown parameter, with one clock offset required for each epoch of GPS observations. The observation equation for double-difference code range measurement is found by twice differencing Equation (9) across two ground stations and two satellites. Explicitly, this is

$$\Delta\nabla p = (|\, \mathbf{r}_{m/b} \,|-|\, \mathbf{r}_{m/i} \,|) - (|\, \mathbf{r}_{r/b} \,|-|\, \mathbf{r}_{r/i} \,|). \tag{10}$$

The double-difference code range measurement is denoted by $\Delta\nabla p$ and the master and remote stations by m and r, respectively. The base and other satellite are indicated by b and i. Unlike the undifferenced code observations, the double-difference code observations do not require the addition of any parameters to the adjustment. Finally, for the double-difference carrier phase measurements the observation equation is

$$\Delta\nabla\Phi = (|\, \mathbf{r}_{m/b} \,|-|\, \mathbf{r}_{m/i} \,|) - (|\, \mathbf{r}_{r/b} \,|-|\, \mathbf{r}_{r/i} \,|) + \Delta\nabla N, \tag{11}$$

where $\Delta\nabla\Phi$ indicates the double-difference phase measurement, and $\Delta\nabla N$ the double-difference phase ambiguity that is included in the adjustment as a parameter. One ambiguity is required for each continuously tracked satellite; should a loss-of-satellite-lock occur, a new ambiguity is required.

4. IMPLEMENTATION

There are a number of hurdles that must be overcome to implement a combined GPS/photogrammetric adjustment, but none is more significant than the sheer amount of software development required. A metric of the effort involved is the more than 85,000 lines of code of which the software currently consists.

4.1. Structure of Combined Adjustment Program

The combined adjustment program has been implemented using GPS and photogrammetric sub-adjustments that are connected in a hierarchical fashion to a root adjustment object. The child adjustments operate in complete isolation from each other and provide only a few generic publicly accessible routines. These routines perform such tasks as calculating parameter approximates, validating adjustment quantities, and, most importantly, processing the least squares observations. It is then the task of the root adjustment object to call these child adjustment routines in the appropriate order. Each child adjustment is, furthermore, responsible for handling the parameters that belong uniquely to it. For example, the photogrammetric adjustment must determine how many unknown interior orientation parameters there are, update these parameters between iterations, and inform the parent adjustment when they have converged. Similarly, the GPS adjustment must do the same for the receiver clock biases. The only parameters that the parent adjustment is responsible for are the unknown positions. The hierarchical adjustment framework is shown in Figure 1. In addition to the photogrammetric and GPS adjustments, this figure shows an additional terrestrial-network child adjustment object that has also been partially implemented.

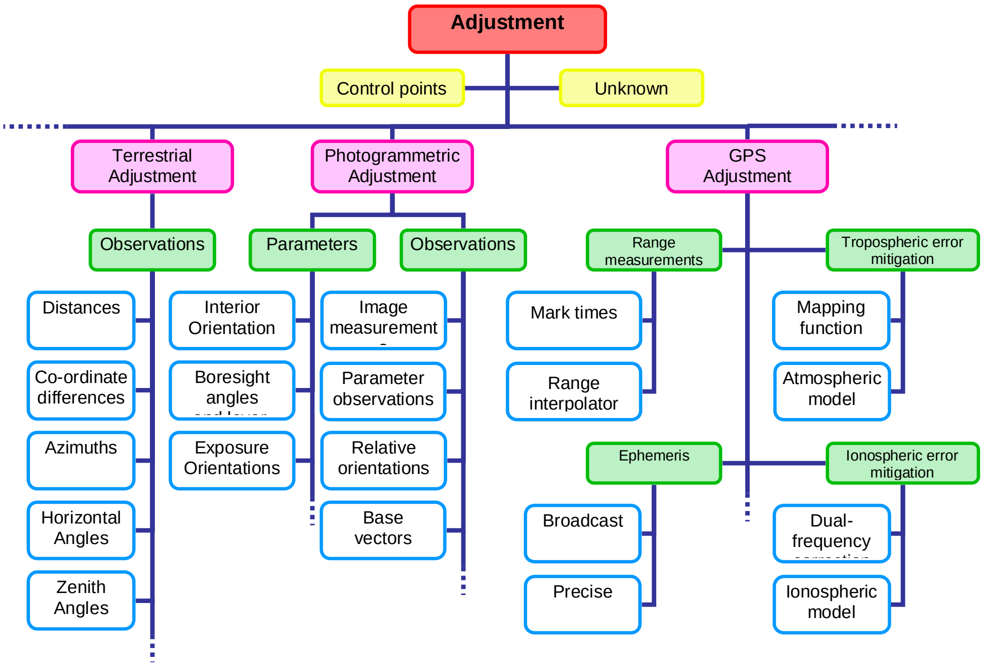

Figure 1: Structure of combined adjustment program

In addition to the tasks given above, the parent adjustment is also responsible for solving the least-squares system of equations. This too, has been made as flexible as possible by delegating the actual numerical least-squares code to a least-squares solution object. This “solver” object contains generic routines that do the following:

1. Add observations to the least-squares system of equations.
2. Solve the least-squares system of equations.
3. Determine the final variance-covariance matrix for the parameters.

Different solvers have been implemented, including one that uses the normal equations and one that uses QR-decomposition. In the former case, observations are added using summation of normals; in the latter case the observations are processed using given’s rotations. Additional notes regarding performance considerations of the current design of the combined adjustment can be found in Ellum (2004).

4.2. GPS ADJUSTMENT NOTES

The GPS processor used in the combined adjustment has a number of idiosyncrasies when compared with other GPS processors. To begin with, since the exposure events don't coincide with GPS measurements, the processor can interpolate measurements between GPS measurement epochs. Polynomial interpolation is used, and tests have shown that linear interpolation causes negligible to non-existent degradation in positioning results. The GPS adjustment has also been designed from the outset so that multiple (i.e., more than two) GPS stations can be used simultaneously. While this, in itself, is not too unusual, it is rather unique that none of the stations need to have fixed co-ordinates. Instead of fixed GPS control, the datum for the entire network can be controlled by information coming from another child adjustment – for example, ground control points that are part of the photogrammetric adjustment, or zenith angles in the terrestrial network adjustment.

It should be emphasised that all the unknown parameters in the combined adjustment, including the GPS specific parameters, are solved for in a batch adjustment. This is in contrast to most GPS processing software, which, even for static periods, uses a Kalman filter operating sequentially in time. The batch adjustment includes both the static *and* kinematic measurement epochs. Even though the adjustment only operates with discrete epochs of GPS data with no time-dependent connecting model, it is still necessary to traverse sequentially through each GPS data file. This is required in order to perform carrier phase smoothing of the code ranges, interpolate observations, detect cycle-slips that cause ambiguities, and track base satellite changes.

5. TESTING

The combined adjustment has been tested by comparing it to the existing technique of position observations. In all tests, the position observations were generated using the adjustment program and same configuration as the combined adjustment, except that the image measurements were not included. The position observations generated as such have been found to have the same or better accuracy as corresponding positions generated by a commercial kinematic processor using the same type of observations.

The comparison of results will primarily be done using the standard deviations of the check point errors. This in acknowledgement of the fact that a mean error – primarily due to unmodelled tropospheric delays – will almost certainly be present in the networks determined using the undifferenced GPS code ranges. Furthermore, for both networks only orthometric heights were available for the check points.

The results presented below supersede results from similar tests reported in Ellum (2004). Due to implementation problems results in that paper should no longer be considered reflective of the performance of the combined adjustment.

5.1. Data description

The data set used for testing was a block of 42 aerial images captured at a photo scale of approximately 1:15,000. Image acquisition was done using a digital camera with a resolution of 4077 × 4092 pixels. Co-ordinates were available for 53 ground points. GPS data at 2Hz was collected on the aeroplane and at 1Hz at a master station located in the centre of the block. The arrangement of the block can be seen in Figure 2.

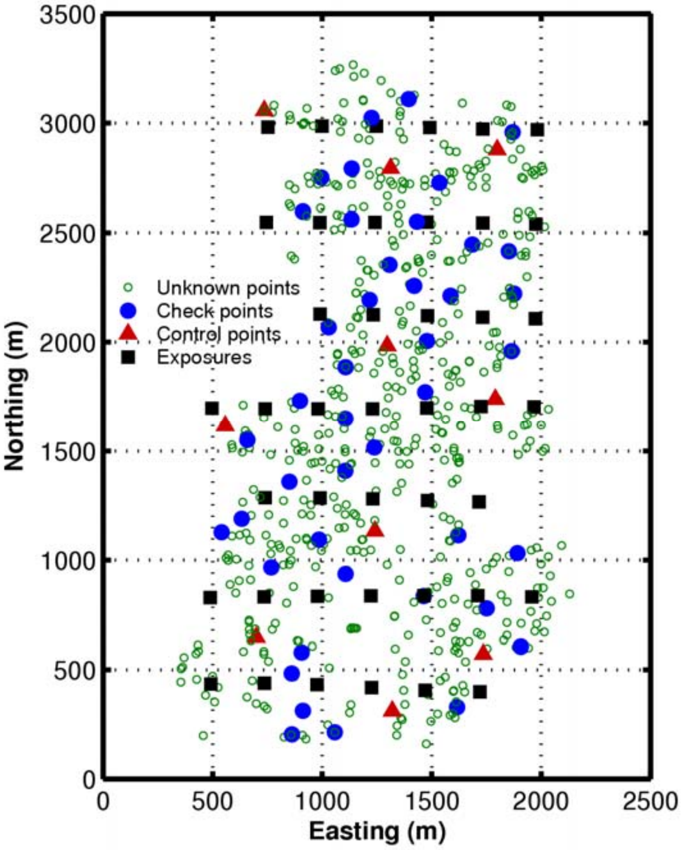

Figure 2: Test field for digital images

5.2. Results

The first adjustments performed with this data set were done to establish the noise level inherent in the network. This noise level, which is, in turn, primarily due to the image measurement noise, was observed using two configurations: a network controlled using ground points, and a network controlled using the best available GPS positions. For the ground controlled network, 10 well-distributed points were selected to act as control and the balance of the points – 43 in total – were used as check points. Figure 2 shows the distribution of these points. For the GPS-controlled network, exposure station position observations were generated by a commercial GPS processor using dual-frequency data. Ambiguities were reported as fixed for all stations. For consistency, the same 43 check points were used to generate the statistics as were used in the ground-controlled adjustment.

The results for these two network configurations are shown in Tables 1 and 2. The results from both configurations indicate that there is about 15cm of horizontal and 35cm of vertical noise in the network. These are, it is believed, the highest-achievable accuracies available from the data and form the basis of comparison for later tests. As noted above, the large difference in the vertical mean errors for the controlled by GPS position observations is because the check points used orthometric heights.

Table 1: Check-point statistics for ground-controlled network

Statistic	Horizontal	Vertical
Mean (m)	0.20	-0.04
Std. Dev. (m)	0.13	0.35

Table 2: Check-point statistics for network controlled using best-available GPS position observations

Statistic	Horizontal	Vertical
Mean (m)	0.28	-40.16
Std. Dev. (m)	0.21	0.37

Undifferenced ranges

The first tests of the combined adjustment were done using undifferenced code ranges. The new combined adjustment is compared against the traditional method of position observations in Tables 3 and 4. Unfortunately, the results from the tests in these tables indicate that the combined adjustment does not appear to offer any improvement in accuracy.

Table 3: Check-point statistics for combined adjustment done using undifferenced code ranges

Statistic	Horizontal	Vertical
Mean (m)	3.97	-53.46
Std. Dev. (m)	0.71	1.51

Table 4: Check-point statistics for undifferenced code range position observations

Statistic	Horizontal	Vertical
Mean (m)	4.04	-53.36
Std. Dev. (m)	0.75	1.47

Tables 5 and 6 show the result when carrier-phase smoothed undifferenced observations were used both in the combined adjustment and to generate position observations. Here, the combined adjustment does offer some improvement in the horizontal accuracy.

Table 5: Check-point statistics for combined adjustment done using smoothed undifferenced code ranges

Statistic	Horizontal	Vertical
Mean (m)	4.11	-53.65
Std. Dev. (m)	0.40	1.34

Table 6: Check-point statistics for smoothed undifferenced code range position observations

Statistic	Horizontal	Vertical
Mean (m)	4.11	-53.18
Std. Dev. (m)	0.78	1.42

Double-difference code ranges

The next set of tests involved the double-differenced code ranges. For unsmoothed ranges, results of which are shown in Tables 7 and 8, the combined adjustment shows a slight increase in accuracy. However, for carrier-phase smoothed ranges, the combined adjustment gives virtually the same results as when position observations are used. Surprisingly, with smoothed ranges both the combined adjustment and position observations provide accuracies that are effectively equivalent to best results possible from network.

Table 7: Check-point statistics for combined adjustment done using double-differenced code ranges

Statistic	Horizontal	Vertical
Mean (m)	1.31	-40.22
Std. Dev. (m)	0.15	0.66

Table 8: Check-point statistics for double-differenced code range position observations

Statistic	Horizontal	Vertical
Mean (m)	1.51	-39.67
Std. Dev. (m)	0.32	0.75

Table 9: Check-point statistics for combined adjustment done using smoothed double-differenced code ranges

Statistic	Horizontal	Vertical
Mean (m)	0.36	-40.78
Std. Dev. (m)	0.17	0.38

Table 10: Check-point statistics for smoothed double-differenced code range position observations

Statistic	Horizontal	Vertical
Mean (m)	0.31	-40.73
Std. Dev. (m)	0.15	0.36

Double-difference carrier-phases and code ranges

The final set of tests used both double-differenced code ranges and carrier-phases. Real (float) ambiguities were estimated in the adjustment. Tables 11 and 12 show the results from these tests. Yet again, the combined adjustment and position observations methods provide near-identical results. Notably, results in both cases are worse than when only smoothed code ranges are used. The cause of this may be that some of the ambiguities were only observed at a few stations, and thus could not be reliably estimated in the adjustment

Table 11: Check-point statistics for combined adjustment done using smoothed double-differenced code ranges and carrier-phases

Statistic	Horizontal	Vertical
Mean (m)	0.54	-40.80
Std. Dev. (m)	0.23	0.37

Table 12: Check-point statistics for smoothed double-differenced code range and carrier-phase position observations

Statistic	Horizontal	Vertical
Mean (m)	0.58	-40.55
Std. Dev. (m)	0.24	0.37

5.3. Analysis

The most obvious observation that can be made from the results presented above is that the combined adjustment offers no real improvement in accuracy to the position observations method. In only two cases was there a noticeable increase in accuracy. This was, admittedly, both unexpected and disappointing; however, the combined adjustment may still have advantages in reliability over the traditional approach, and more testing is required to confirm or discard this hypothesis.

Another surprising observation is that regardless of the technique used, simply using smoothed doubly-differenced code ranges gave check-point accuracies that were virtually the same as those available from the most well-controlled network configurations. This indicates that difficult and possible unreliable ambiguity fixing may not be necessary at all, and that cheaper single-frequency receivers may be sufficient for the most commonly encountered network configurations. Such results run contrary to commonly help beliefs.

6. OUTLOOK

The testing of the combined adjustment done for this paper has not been sufficiently detailed to enable concrete conclusions to be drawn regarding the performance of the method. At this point, it appears as if the combined adjustment does not offer improved accuracy over the traditional technique of integrating GPS and photogrammetry. However, the combined adjustment still has the benefits of streamlined processing and flexible use of GPS measurements outlined in Section 2.3 and so even without improved accuracy the use of the combined adjustment may still be advantageous. Additionally, the important question of whether the technique provides improved reliability has not yet been addressed.

A number of improvements are possible to the combined adjustment. These improvements should improve its accuracy. Currently, for example, a base satellite change introduces an entire set of new ambiguities into the adjustment. However, providing a cycle slip does not occur as the base satellite is changing it is possible to add constraint equations to the adjustment that, in effect, transfer the old ambiguities to the new base satellite (Radovanovic, 2002). This would enable any static sessions at the beginning and end of the flights to be used to aid ambiguity resolution. Another improvement is to enable integer ambiguity resolution in the adjustment. Code to do this has been implemented, but not yet tested.

Additional testing in other network configurations is also required. Obvious candidates include networks where:

- less than four satellites are used
- the GPS master station(s) are not fixed
- different and non-standard block configurations are used

ACKNOWLEDGEMENTS

Joe Hutton and Dr. Mohammed Mostafa at Applanix, Inc. are thanked for providing the test data set. Funding for this research was provided by the Killam Trusts and the Natural Sciences and Engineering Research Council of Canada (NSERC).

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BIOGRAPHICAL NOTES

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