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Geo-Resources and Territory Department

An automatic analytical procedure for searching corresponding feature points in a cadastral map

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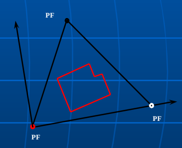
Introduction

- 1987: the Italian Cadastre Authority promoted the computer based management of the field surveys
- 1988: every new cadastral field survey must be referred to at least 3 fiducial points properly chosen (⇨ Fiducial Point Network definition)



- 2004: the national cadastre archive contains more than 30.000.000 measurements collected

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Problems

- The fiducial point network accuracy is generally inadequate. Most of the FP coord. are still floating
- Current (EDM, GPS) accurate surveys outline the discrepancies of the existing cadastral maps

Proposal

⇨ Use of the huge field survey data base …
- to improve the fiducial point network accuracy
- to update the cadastral map

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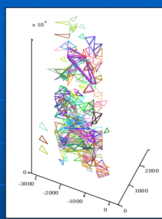
The Procrustes Method

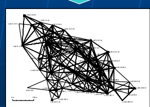
A – FP network re-definition by aggregation and adjustment of the field surveys

B – Digital map updating by insertion of the new surveyed entities (parcels and buildings)

C – Simultaneous adjustment of FPs and geometric entities

Details: *Beinat & Crosilla – ZfV, 5/2003*





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One last obstacle …

For the full automatic processing, the vertex-to-vertex topologic correspondence must be know!

➸ This condition is **TRUE** for the fiducial point network

Every FP is rigorously and uniquely identified at national wide level by rigid naming rules

➸ This condition is **FALSE** for the surveys of the parcel vertices

The surveyor can give to the parcel vertices an arbitrary id. code. Most of the existing 300.000 digital (r/v) maps have no topology.

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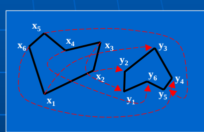
The analytical automatic detection of point correspondences

How identify topologies for:

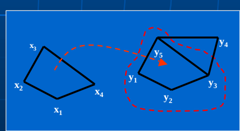
1 - Comparison

Same shapes, different scales, arbitrary orientations, no topology



2 - Inclusion

One shape completely included into a general one, different scales, arbitrary orientations, no topology



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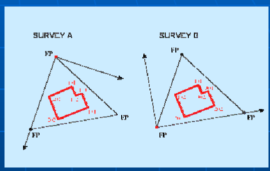
Comparison problem

- Two or more surveys of the same cadastral entity
- Various surveys have different vertex names or no names
- Every survey refers to its own different reference system
- There is no topology, only the vertex coordinates are available



Rilievo A				
Vertice	101	102	103	104
Coordinata X	73,22	95,97	82,94	120,87
Coordinata Y	80,87	100,22	115,39	147,97

?

Rilievo B				
Vertice	201	202	301	302
Coordinata X	151,76	188,04	177,59	158,03
Coordinata Y	113,49	39,44	88,34	84,16

Comparison solution

Given the 2 coordinate matrix X and Y, the aim is to define the permutation matrix P, by which we define an optimal similarity transformation that fits X over Y (a.k.a. "labelling problem").

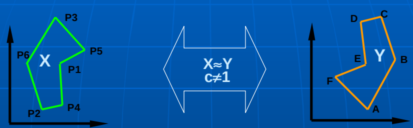
$$\min_{Q,c,t,P} \frac{1}{n} \left\| Y - c \cdot QXP - th^T \right\|^2$$

*Where **Q** is the rotation matrix, **t** the translation, **h** a unitary vector, n the number of points, and **c** the global scale factor.*

To compute P we implemented and adapted an analytical method due to Umeyama (1988).

Another direct geometrical solution will be presented at the next ISPRS meeting in Istanbul (B, C & S; 2004).

Comparison solution example



X Coordinates						
Vertex	P1	P2	P3	P4	P5	P6
Coord. x	61,4	39,4	55,6	64,4	91,9	21,5
Coord. y	72,4	16,6	128,7	22,6	89,5	72,9

Y Coordinates						
Vertex	A	B	C	D	E	F
Coord. x	68,8	102,0	85,0	60,0	66,1	29,1
Coord. y	14,0	74,1	126,1	120,1	68,3	53,7

Adjacency matrix A_X

	P1	P2	P3	P4	P5	P6
P1	0	60,01	56,53	49,92	34,92	39,89
P2	sym	0	113,23	25,71	89,78	59,08
P3	sym	sym	0	106,43	53,44	65,37
P4	sym	sym	sym	0	72,27	66,11
P5	sym	sym	sym	sym	0	72,32
P6	sym	sym	sym	sym	sym	0

Adjacency matrix A_Y

	A	B	C	D	E	F
A	0	68,64	113,23	106,43	54,32	56,14
B	sym	0	54,70	62,28	36,34	75,73
C	sym	sym	0	25,70	60,81	91,48
D	sym	sym	sym	0	52,16	73,24
E	sym	sym	sym	sym	0	39,81
F	sym	sym	sym	sym	sym	0

Comparison solution example

Spectral Decomposition of the Adjacency Matrices A_x and A_y

$\mathbf{D}_X = \text{diag}\,(327{,}44 ; 157{,}43 ; 71{,}35 ; 50{,}96 ; 26{,}42 ; 21{,}45)$ $\quad$ $\mathbf{D}_Y = \text{diag}\,(329{,}62 ; 157{,}97 ; 72{,}06 ; 50{,}86 ; 27{,}42 ; 21{,}29)$

$$\bar{\mathbf{U}}_X = \begin{pmatrix} 0{,}31 & 0{,}08 & 0{,}12 & 0{,}57 & 0{,}62 & 0{,}40 \\ 0{,}43 & 0{,}55 & 0{,}07 & 0{,}25 & 0{,}27 & 0{,}59 \\ 0{,}48 & 0{,}58 & 0{,}21 & 0{,}57 & 0{,}22 & 0{,}03 \\ 0{,}40 & 0{,}47 & 0{,}26 & 0{,}15 & 0{,}34 & 0{,}62 \\ 0{,}40 & 0{,}32 & 0{,}62 & 0{,}22 & 0{,}53 & 0{,}07 \\ 0{,}38 & 0{,}00 & 0{,}69 & 0{,}45 & 0{,}27 & 0{,}30 \end{pmatrix} \qquad \bar{\mathbf{U}}_Y = \begin{pmatrix} 0{,}48 & 0{,}58 & 0{,}21 & 0{,}58 & 0{,}18 & 0{,}00 \\ 0{,}37 & 0{,}06 & 0{,}67 & 0{,}38 & 0{,}36 & 0{,}33 \\ 0{,}43 & 0{,}55 & 0{,}02 & 0{,}27 & 0{,}24 & 0{,}06 \\ 0{,}40 & 0{,}40 & 0{,}29 & 0{,}13 & 0{,}36 & 0{,}61 \\ 0{,}31 & 0{,}09 & 0{,}03 & 0{,}57 & 0{,}64 & 0{,}37 \\ 0{,}41 & 0{,}33 & 0{,}63 & 0{,}29 & 0{,}46 & 0{,}01 \end{pmatrix}$$

$$\mathbf{Z} = \bar{\mathbf{U}}_X \bar{\mathbf{U}}_Y^T = \begin{pmatrix} 0{,}6777 & 0{,}7915 & 0{,}7385 & 0{,}7545 & \mathbf{0{,}9955} & 0{,}7171 \\ 0{,}7576 & 0{,}6462 & \mathbf{0{,}9981} & 0{,}9586 & 0{,}7432 & 0{,}6352 \\ \mathbf{0{,}9987} & 0{,}6749 & 0{,}7700 & 0{,}7093 & 0{,}7008 & 0{,}8097 \\ 0{,}6881 & 0{,}7557 & 0{,}9553 & \mathbf{0{,}9991} & 0{,}7307 & 0{,}7193 \\ 0{,}7511 & 0{,}9028 & 0{,}6078 & 0{,}7687 & 0{,}6884 & \mathbf{0{,}9936} \\ 0{,}6515 & \mathbf{0{,}9919} & 0{,}5615 & 0{,}7078 & 0{,}7013 & 0{,}8708 \end{pmatrix}$$

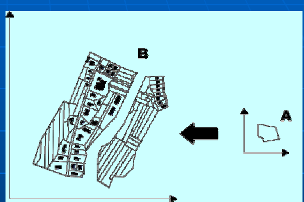
P =

	A	B	C	D	E	F
P1	0	0	0	0	1	0
P2	0	0	1	0	0	0
P3	1	0	0	0	0	0
P4	0	0	0	1	0	0
P5	0	0	0	0	0	1
P6	0	1	0	0	0	0

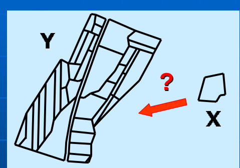
(a final shape test validates the solution)

Inclusion problem

- The survey is entirely contained into the general cadastral map
- Map and survey have different vertex names
- Map and survey have their own different reference systems
- There is no topology, only the vertex coordinates are available



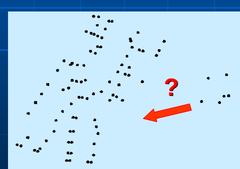
Inclusion problem



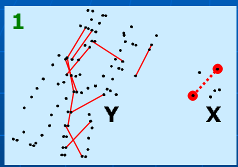
How we figure it …

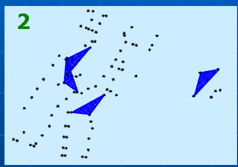
But …

what it actually consists (without topology)



Inclusion solution

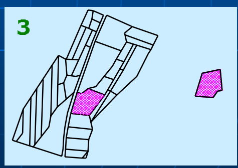




1 – The max. element of the adjacency matrix of X is set

2 – A "suitable" triangle is build in X, and seek out in Y

3 – The kernel is expanded to the rest of the points of X

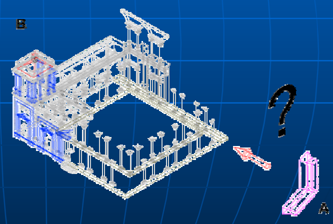


Conclusions

A direct method to detect point-to-point correspondences between two geometric configurations represented by coordinate matrices has been described.

The solution provided is for the case of comparison (similar shapes), and for the case of inclusion (one shape completely contained into another).

As well as for digital mapping, suitable applications are also for CAD/CAM and GIS automation procedures.



Thank-you
for your kind attention